# Beyond A*:

## Speeding up pathfinding through hierarchical abstraction

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# Motivation

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- Myth: Pathfinding is a solved problem.
  - Reality: Pathfinding is not solved!
    - A* underlies many pathfinding systems but...
    - Many open problems persist!

# Problem: Efficient pathfinding in very large environments

Image: World of Warcraft (Blizzard Entertainment)

# Problem: Dynamic terrain

Image: Company of Heroes (Relic Entertainment)

# Problem: Different unit classes

Image: Red Alert 3 (Electronic Arts)

# Problem: Cooperative Pathfinding

Image: Settlers: Heritage of Kings (Blue Byte Software)

# Rest of this talk

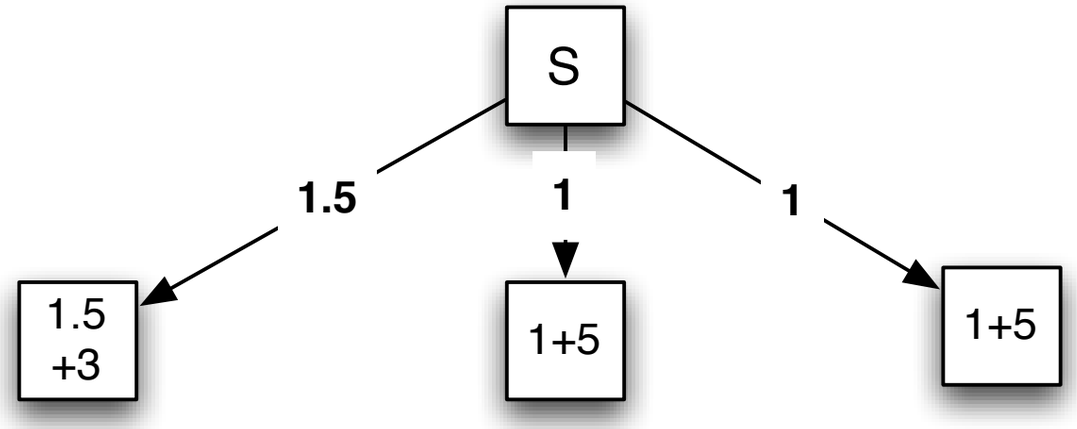
- Some background on A*
- Then, a story in 3 parts:
  - Part 1: Hierarchical pathfinding
  - Part 2: Dealing with diversity
  - Part 3: Probabilistic roadmaps

# Background

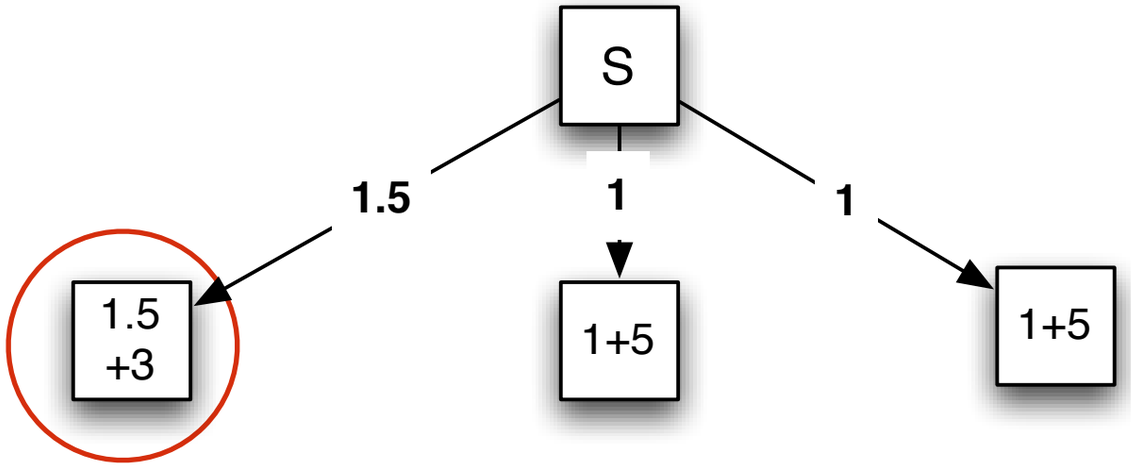
# A* In Brief

- [Hart, Nilsson & Raphael, 1968]
- Heuristic search
- **f(n) = g(n) + h(n),** where:
  **g(n)** = distance so far
  **h(n)** = estimate remaining distance
- Complete + Optimal

# A* example



# A* example



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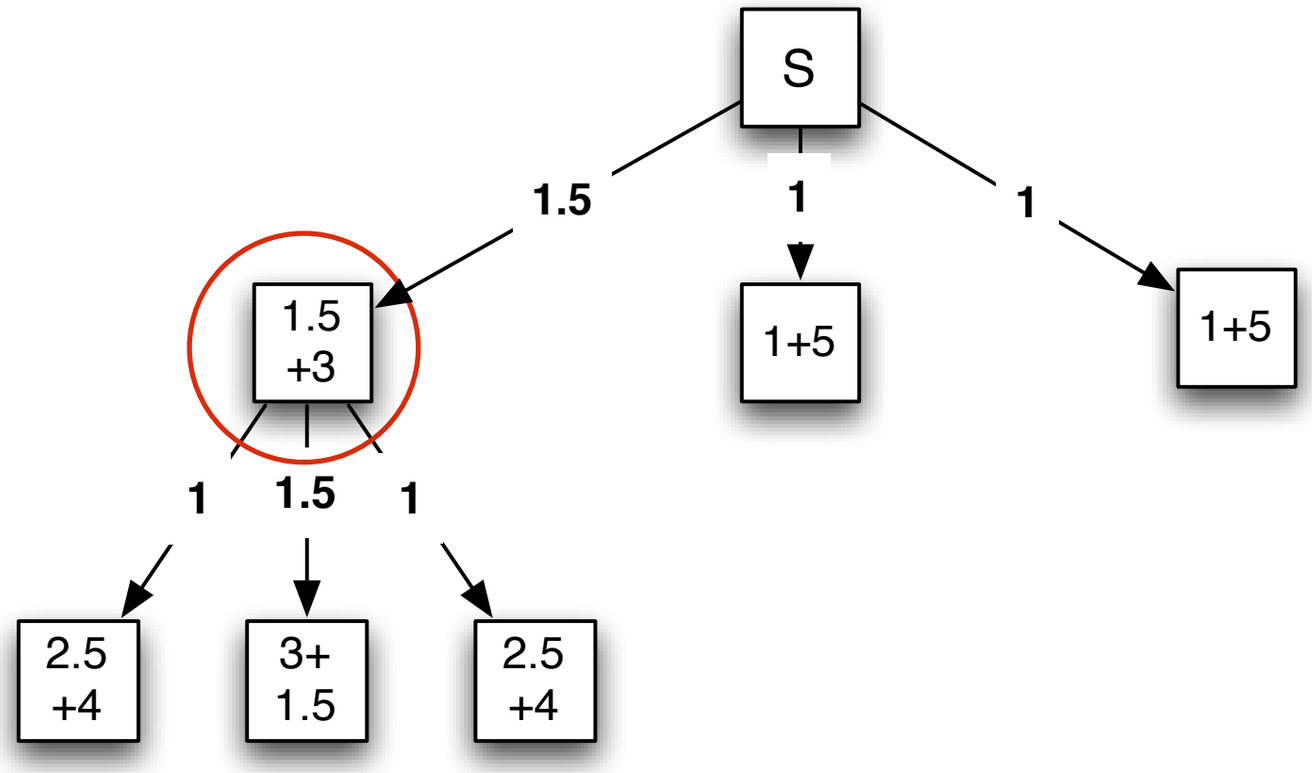

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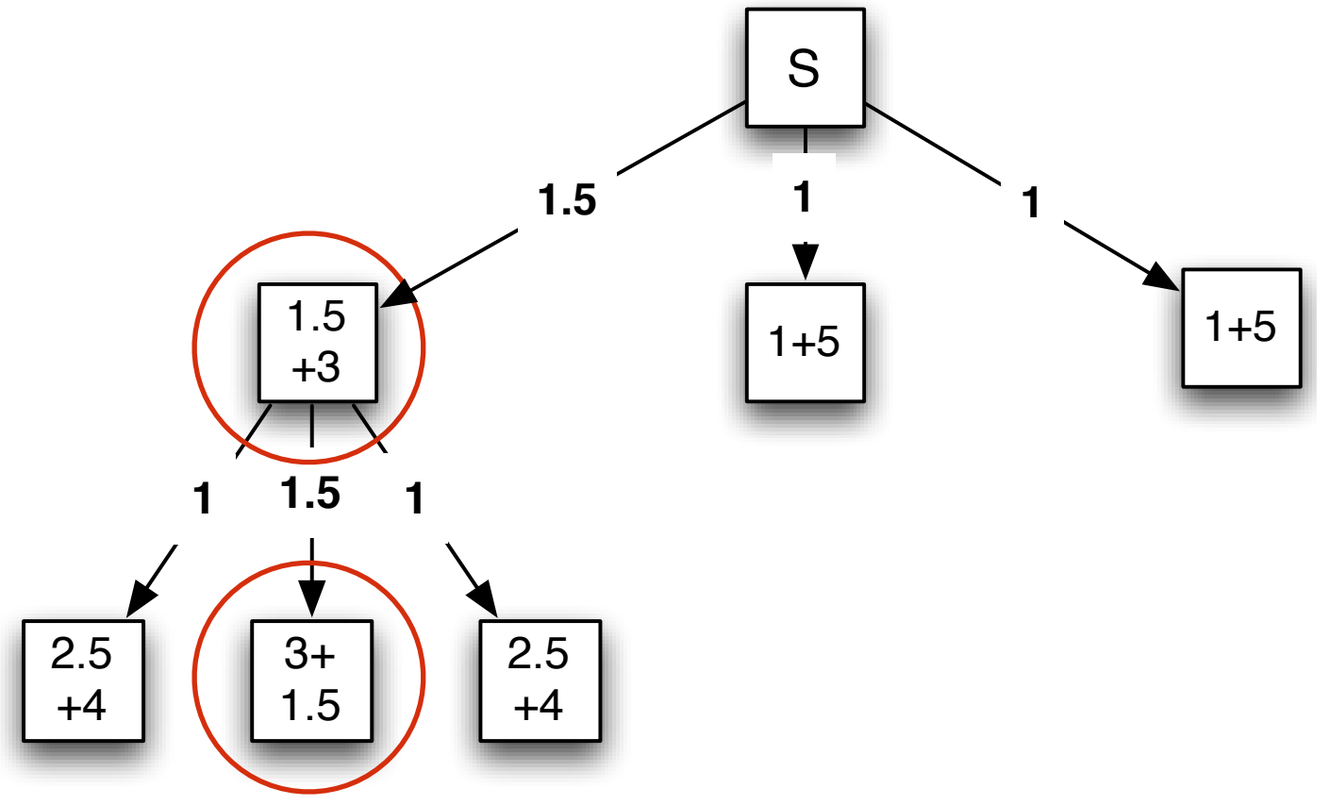

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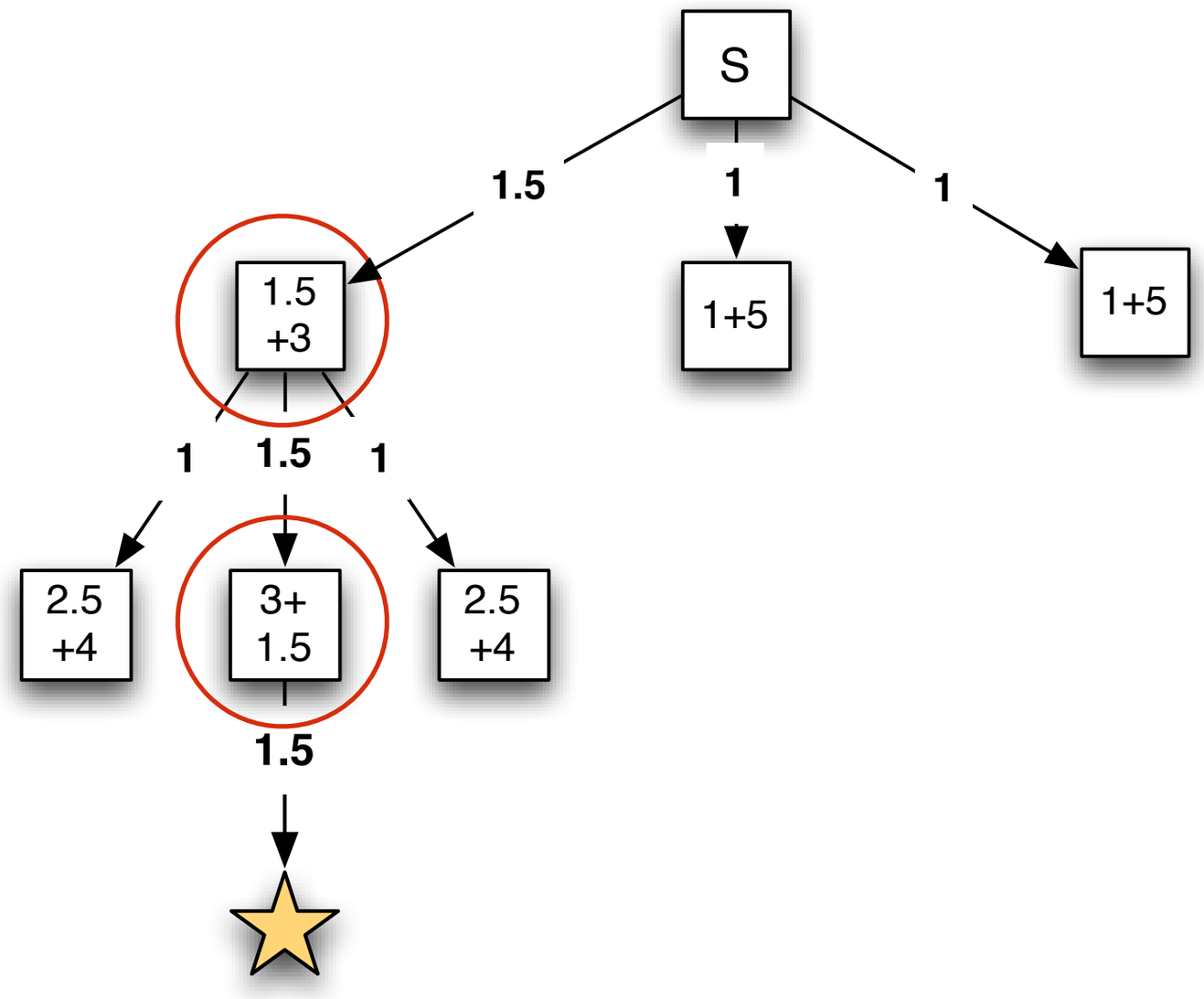
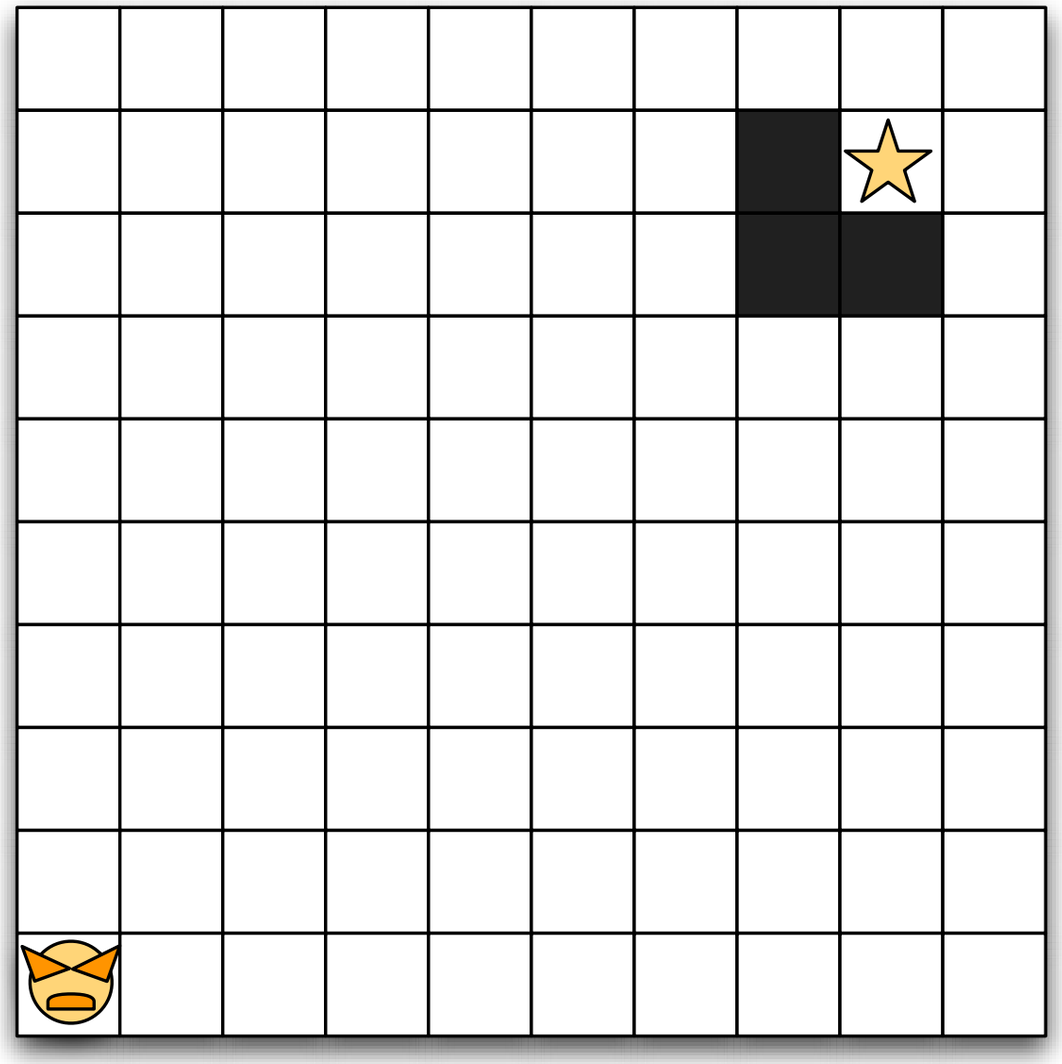


# The problem with A*

- Time complexity (nodes expanded)
- Memory overhead (stuff to remember)
- Good heuristics not enough.

# A* Example



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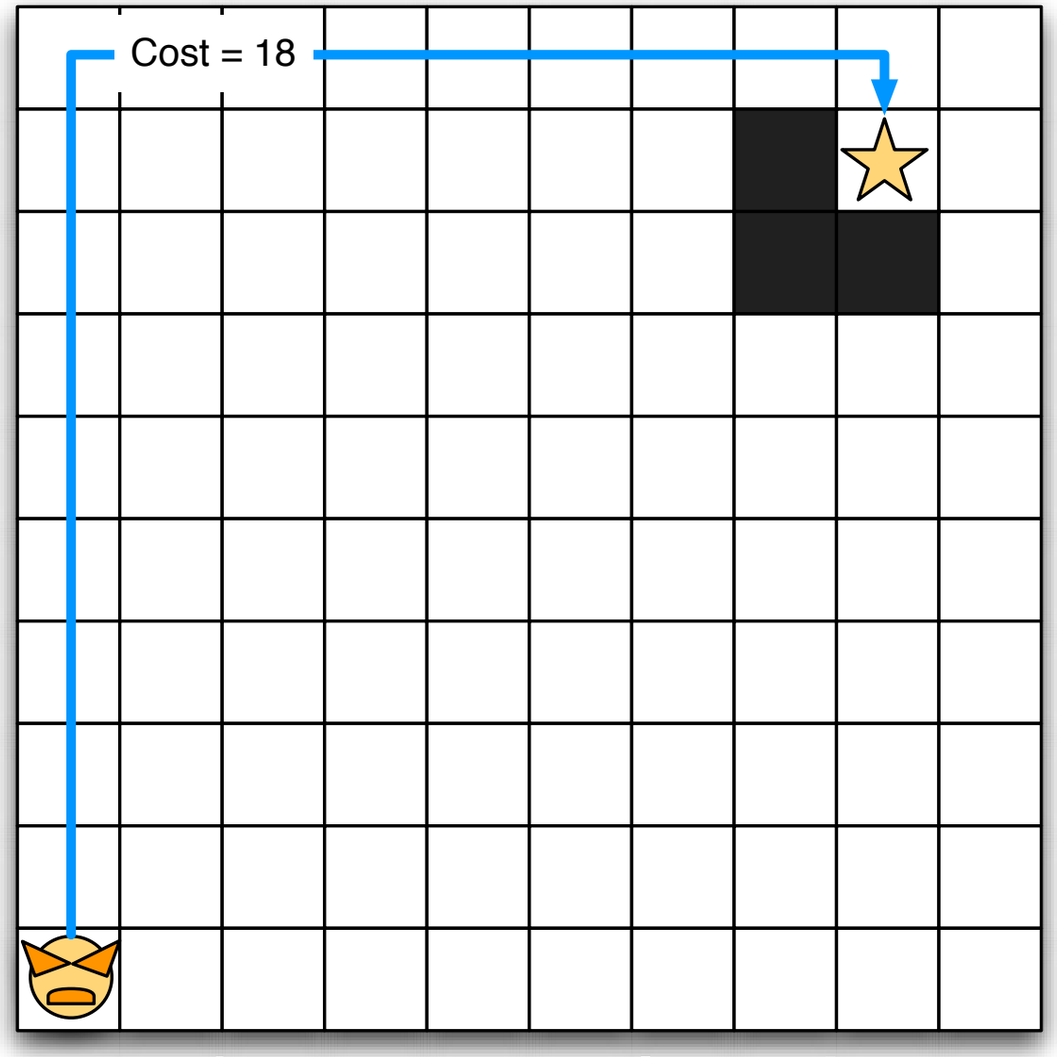
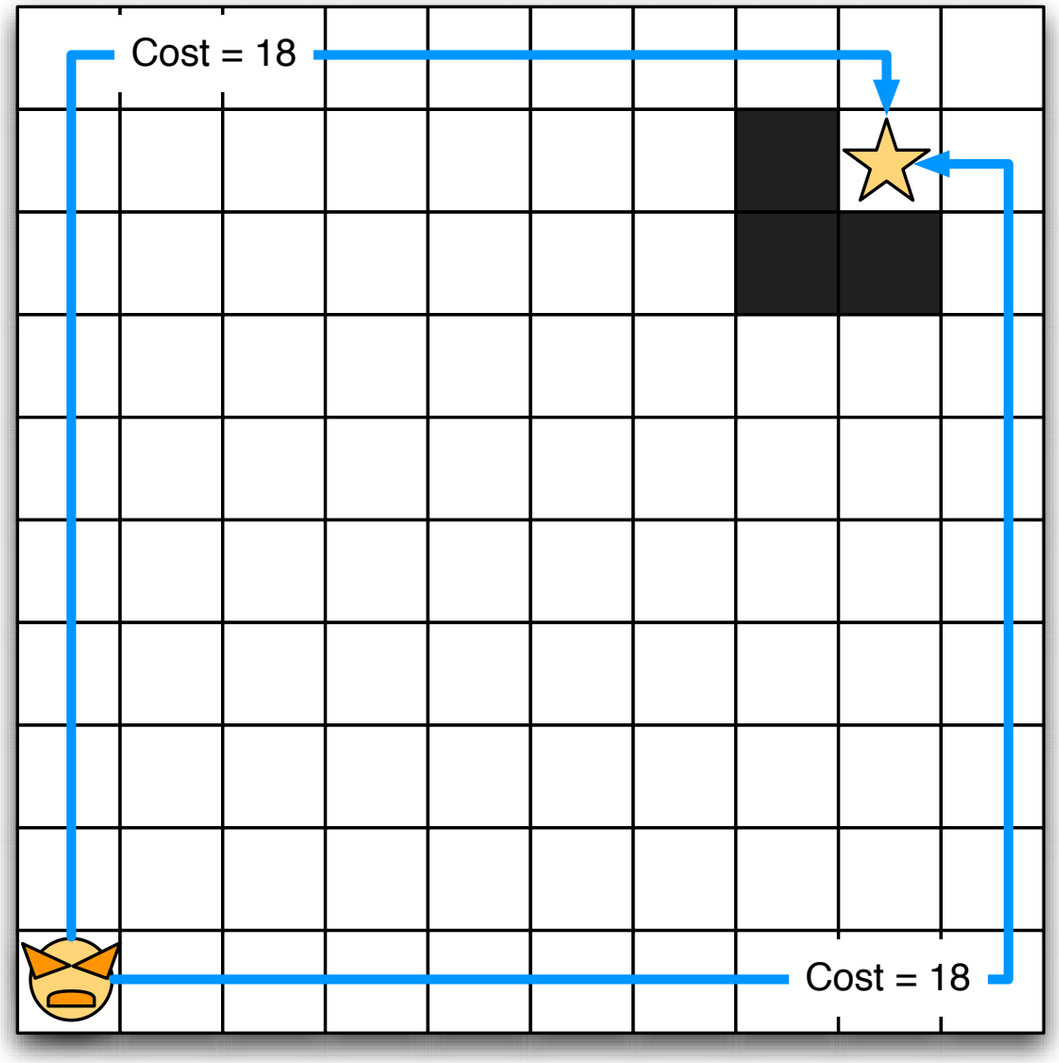


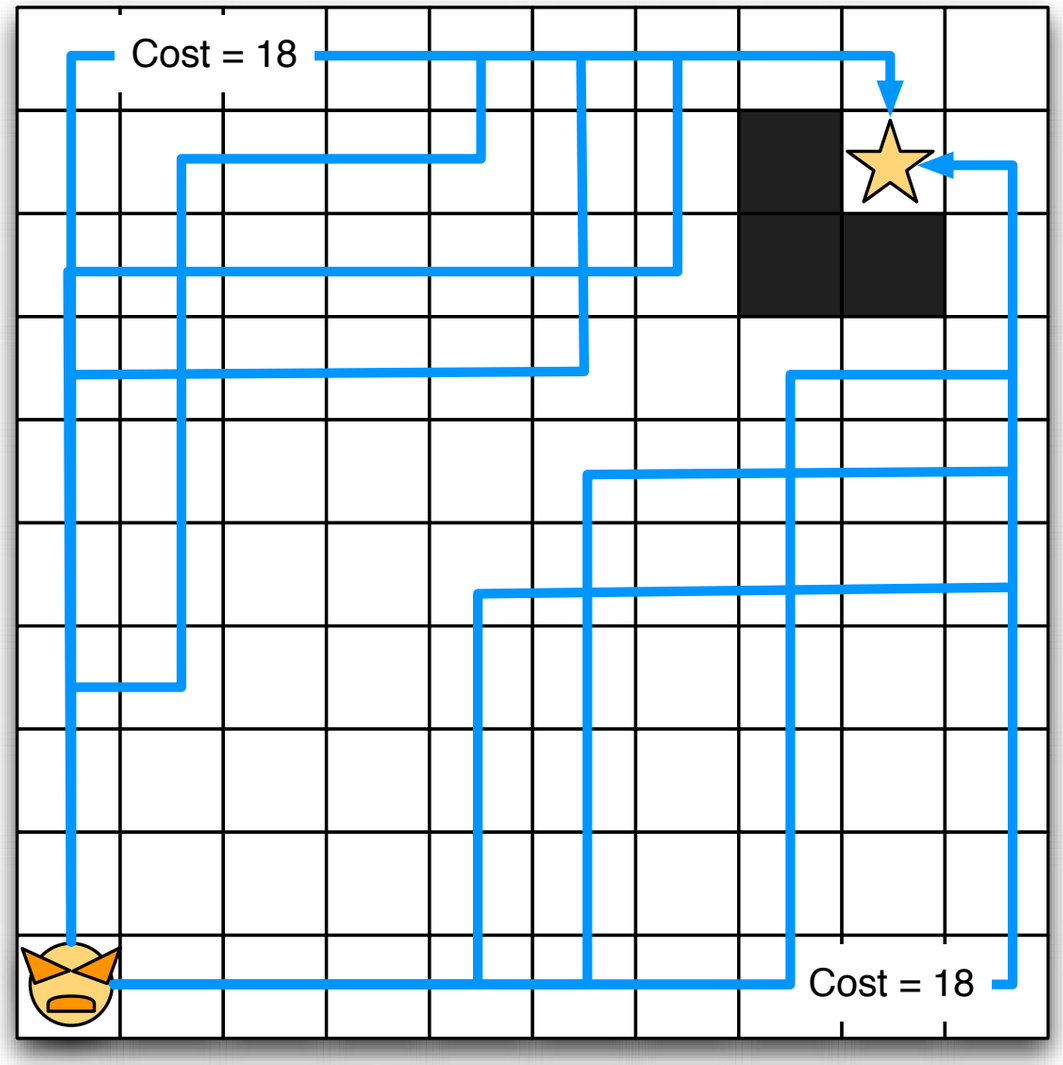

An optimal path
(straight-line movement only)

# A* Example




Another optimal path

# A* Example




Lots of optimal paths
(and many more exist!)

# A* Example

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 18 | 18 | 18 | 18 | 18 | 18 | 18 | 18 | 18 | |
| 16 | 16 | 16 | 16 | 16 | 16 | 16 | | ★ | 18 |
| 16 | | | | | | 16 | | | 18 |
| 16 | | | | | | 16 | 16 | 16 | 18 |
| 16 | | | | | | | | 16 | 18 |
| 16 | | | | | | | | 16 | 18 |
| 16 | | | | | | | | 16 | 18 |
| 16 | | | | | | | | 16 | 18 |
| 16 | | | | | | | | 16 | 18 |
| | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 18 |

A* expands: all green tiles + some white

# Part 1:
# Hierarchical pathfinding

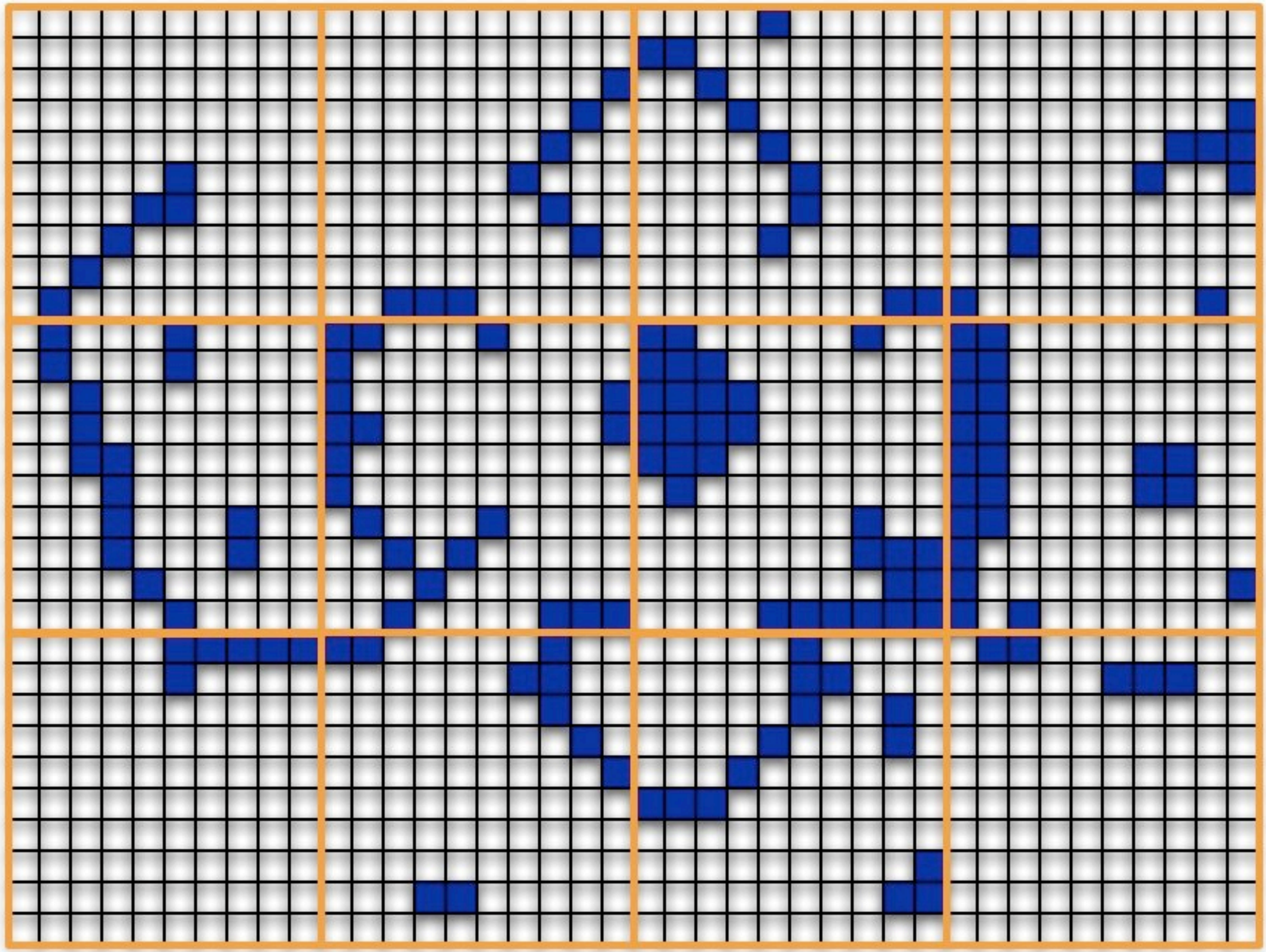
# Hierarchical pathfinding

- Build a smaller, approximate search space.
- Area-based vs. Tile-based pathfinding.
- Factored problems are easy!
- Faster + lower memory requirements.

# HPA* Overview

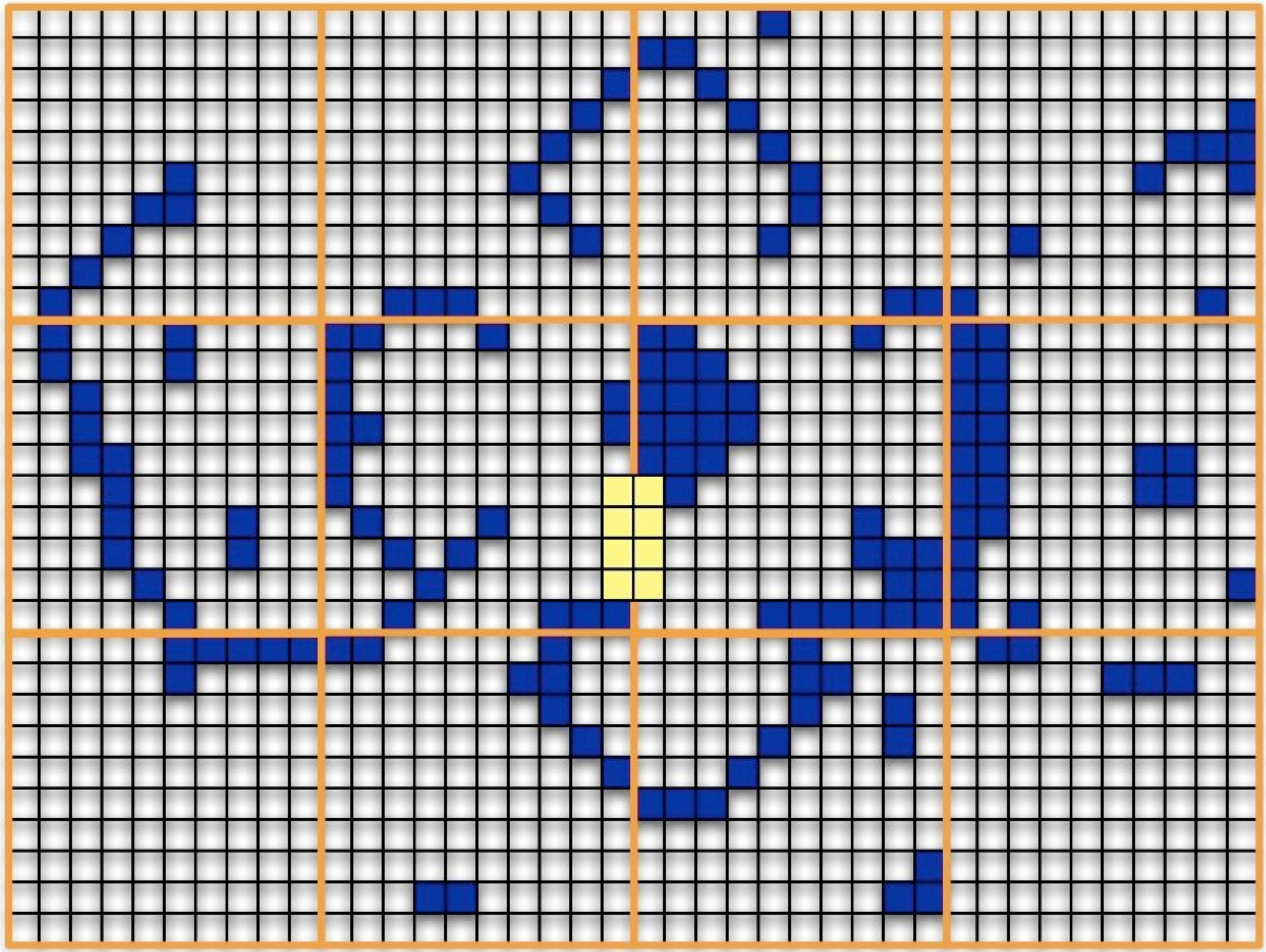
- [Botea, Müller & Schaeffer, 2004]
- Pre-processing:
  - Build abstract graph (Clusters and Entrances)
- Run-time processing:
  - Insert start and goal into abstract graph
  - Find abstract solution
  - Refine if necessary (or use cached info)

Map: Temple Prime (Red Alert 3)



Split map into square clusters (e.g. 10x10)

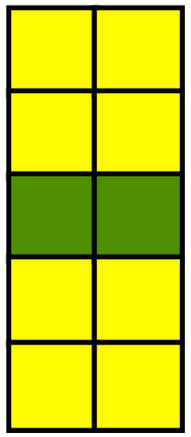
Image: Adi Botea



# Identify entrances between adjacent clusters

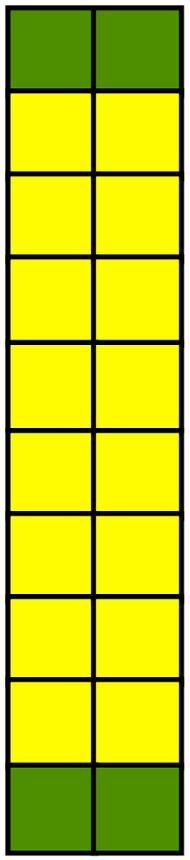
Image: Adi Botea

Small entrance (length < 6)



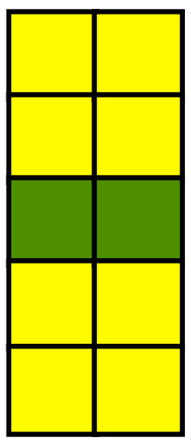
1 central transition point

Large entrance (length >= 6)

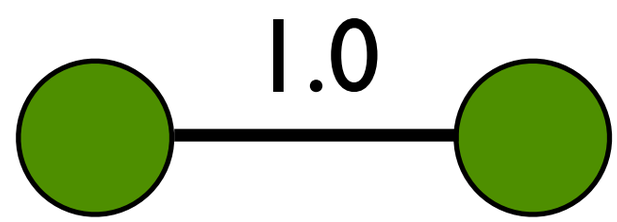


2 perimeter transition points

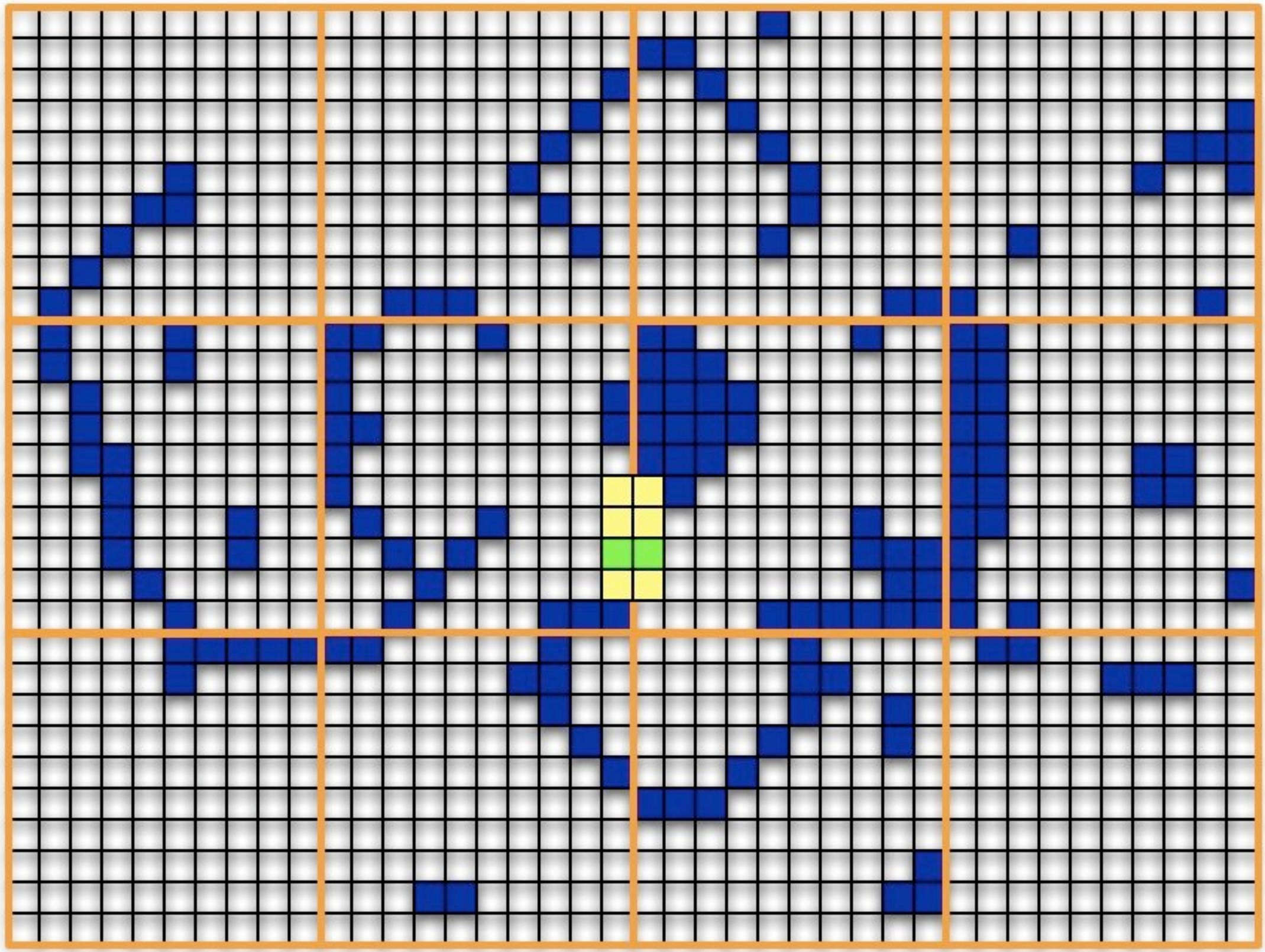
Transition point =
2 abstract nodes + 1 abstract (inter) edge



Entrance

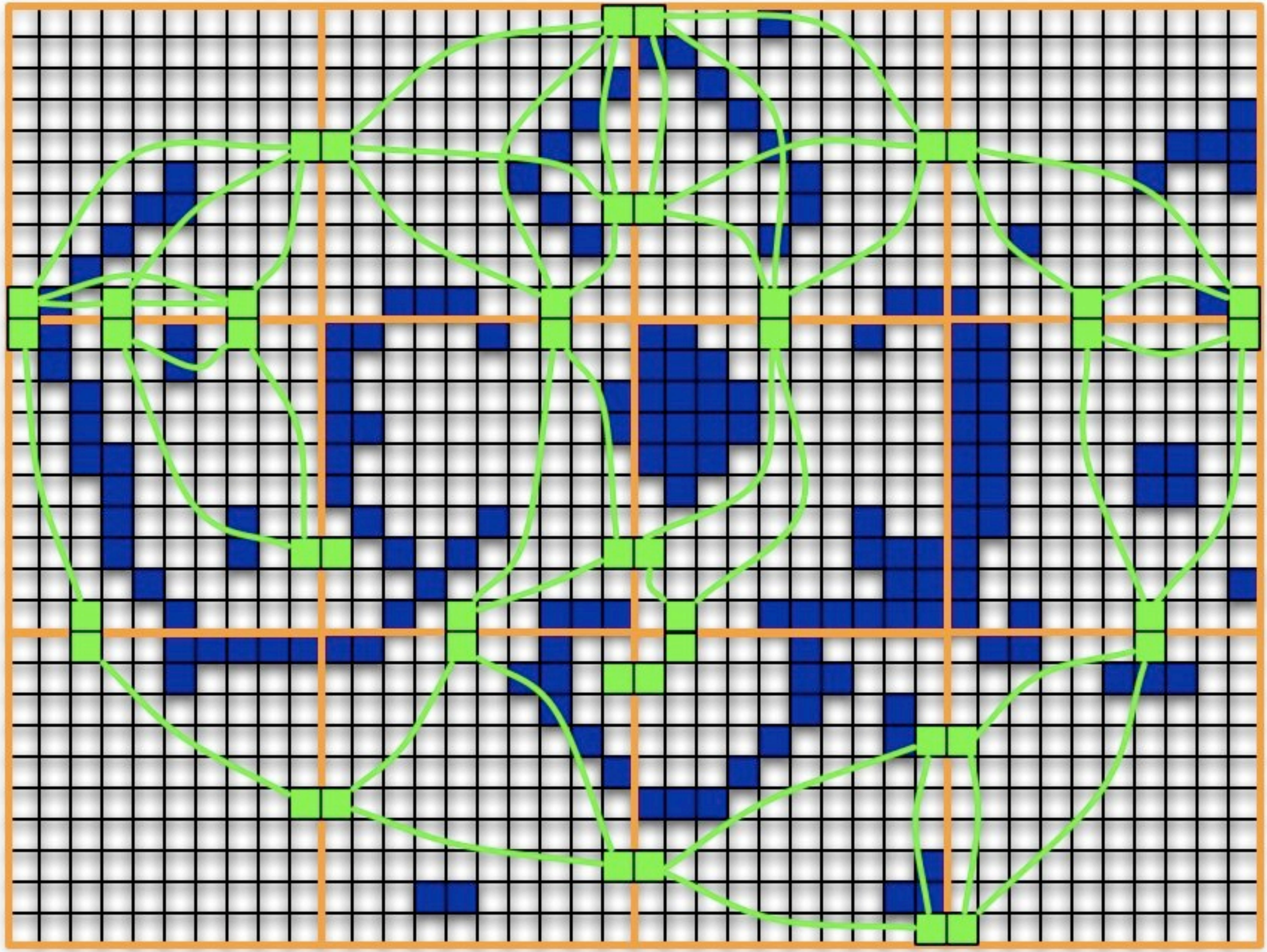



Abstract graph



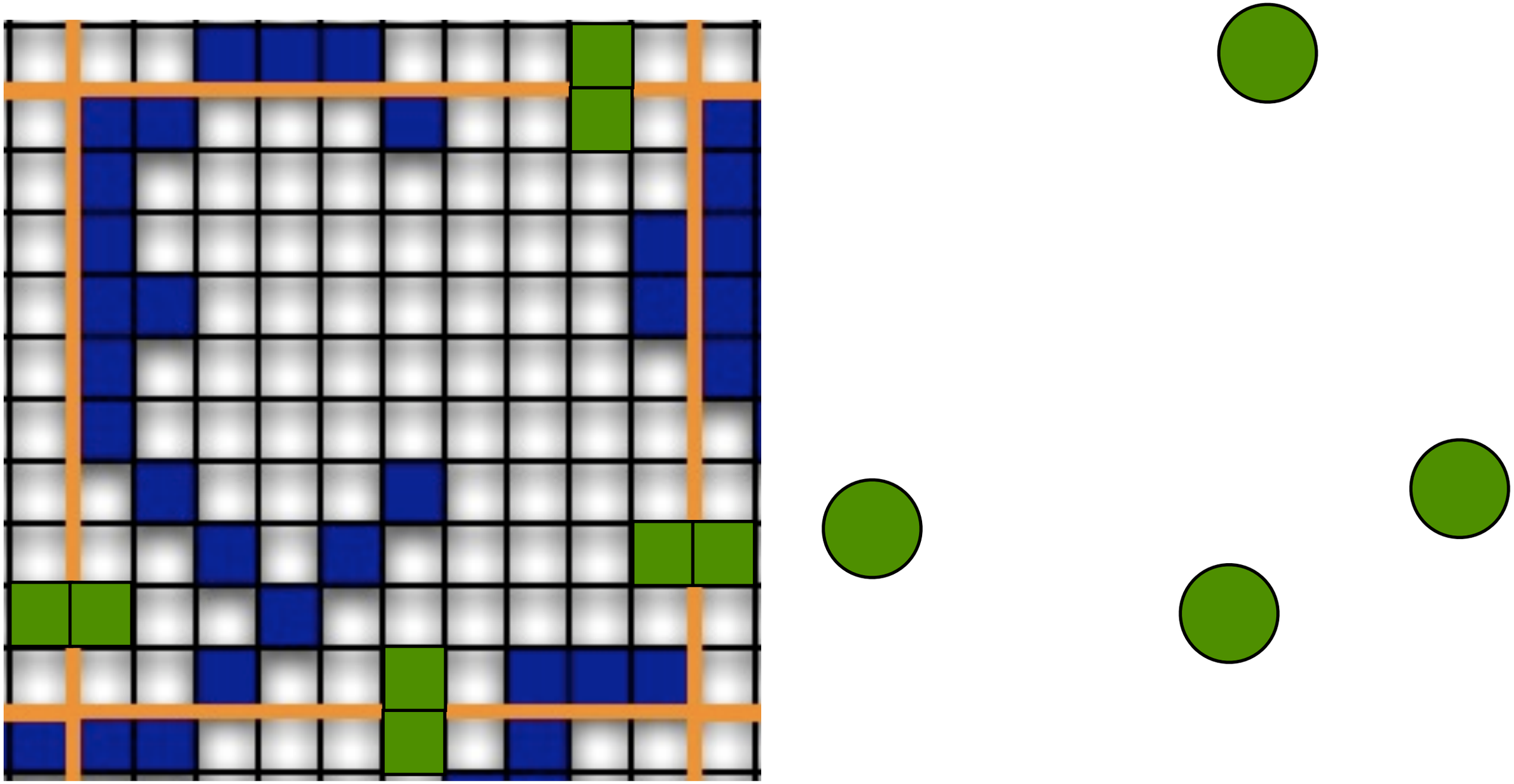
# Select transition points for each entrance

Image: Adi Botea

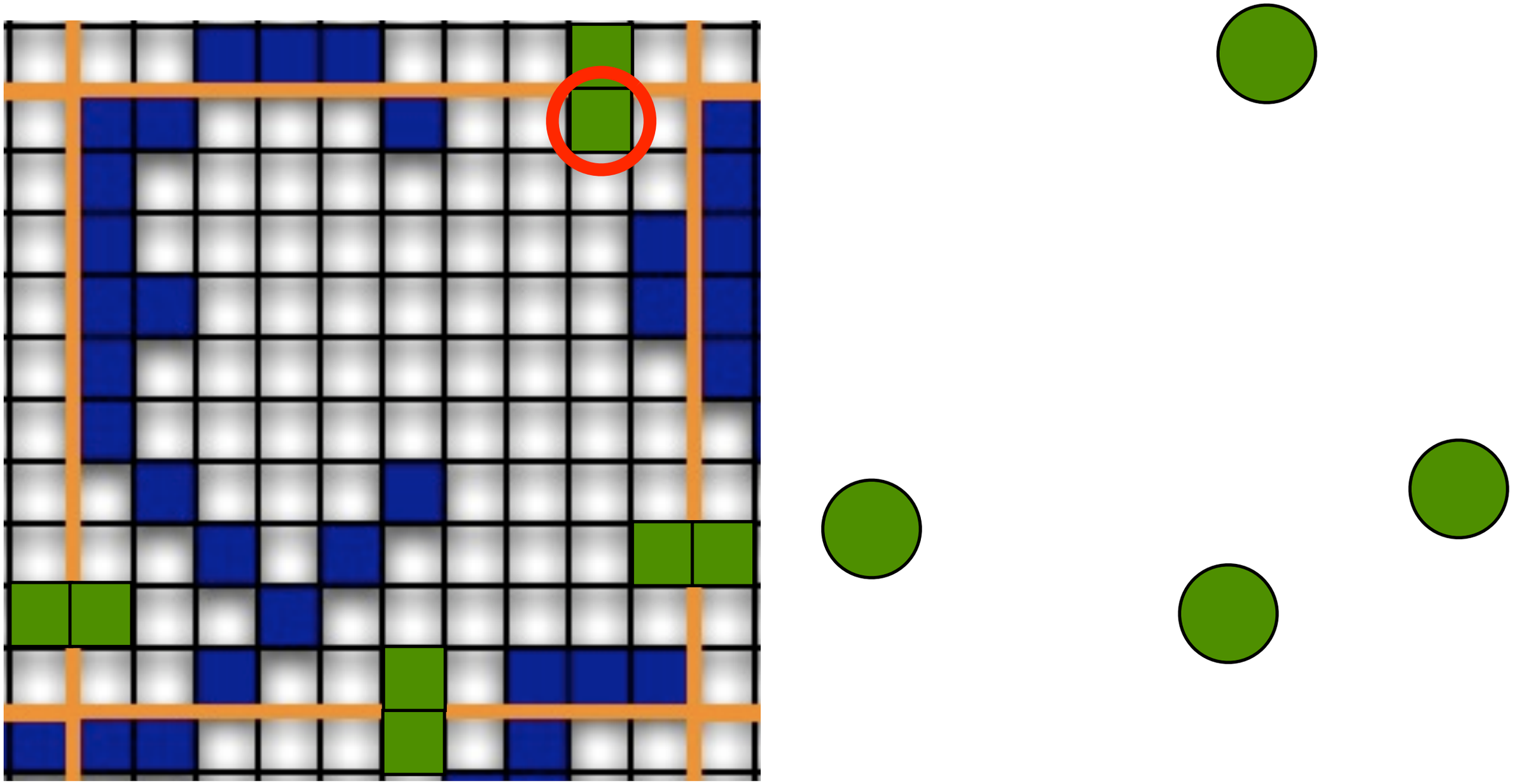


# Connect abstract nodes inside the same cluster

Image: Adi Botea



Intra-edges (cost = path length) added to abstract graph



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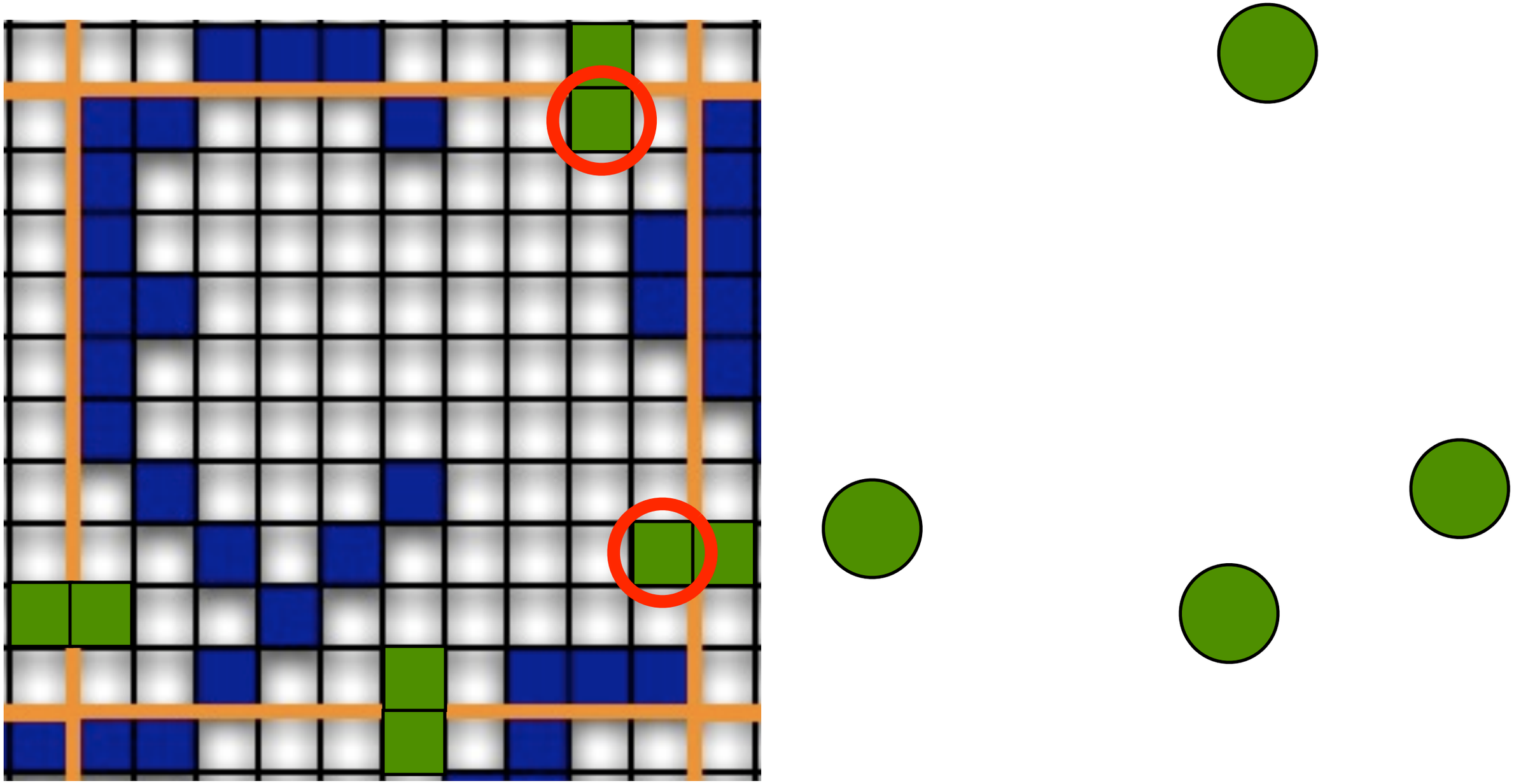

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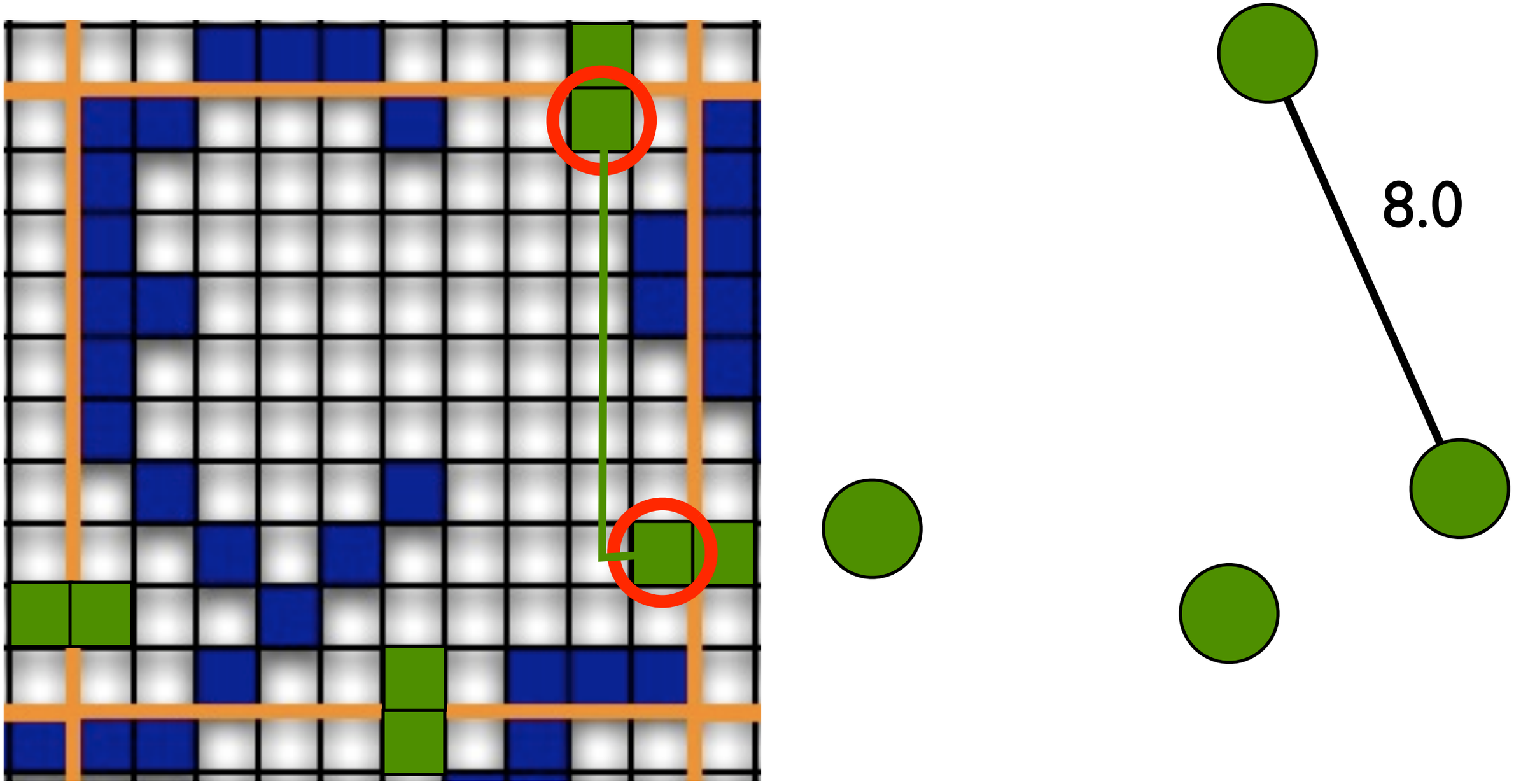


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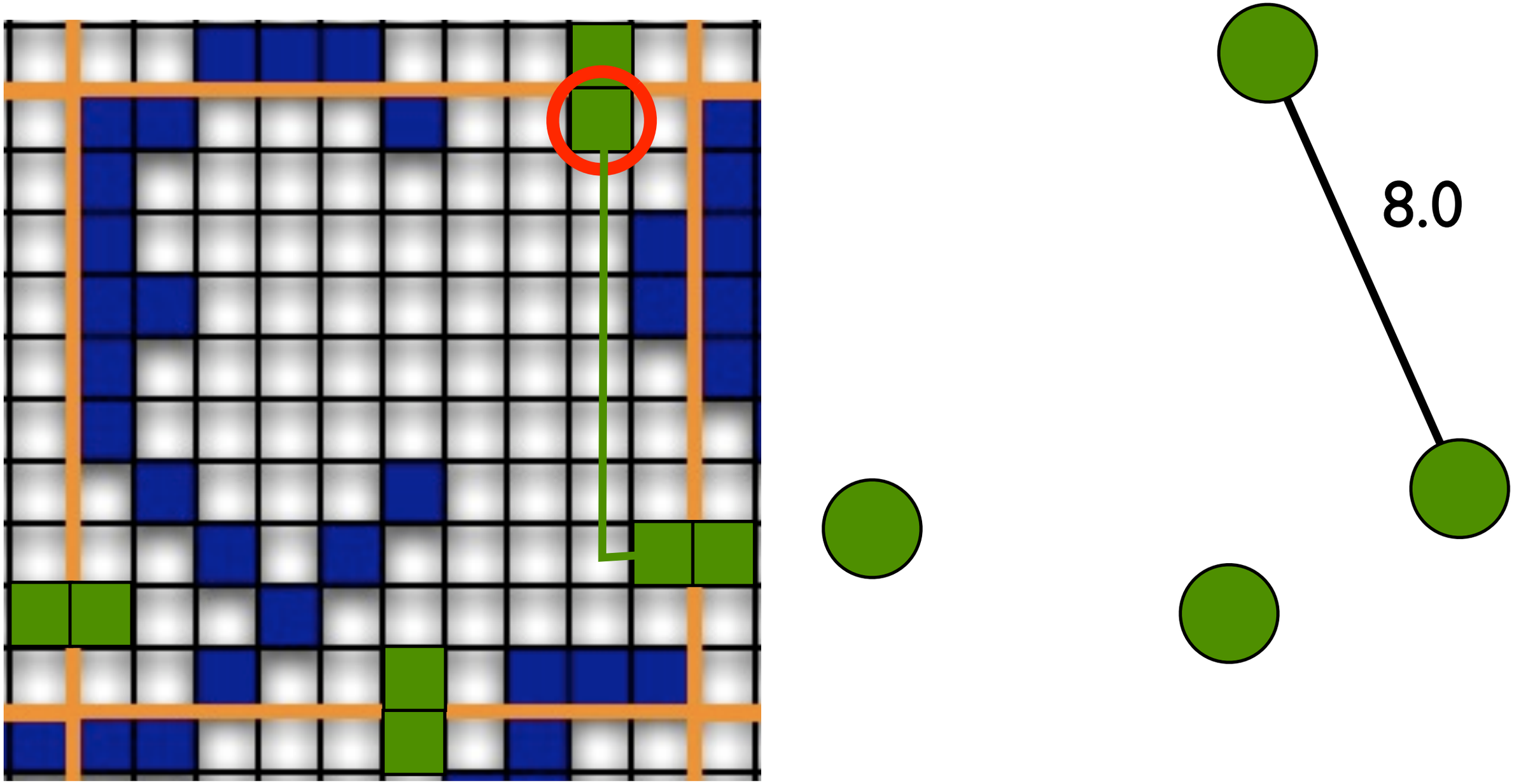


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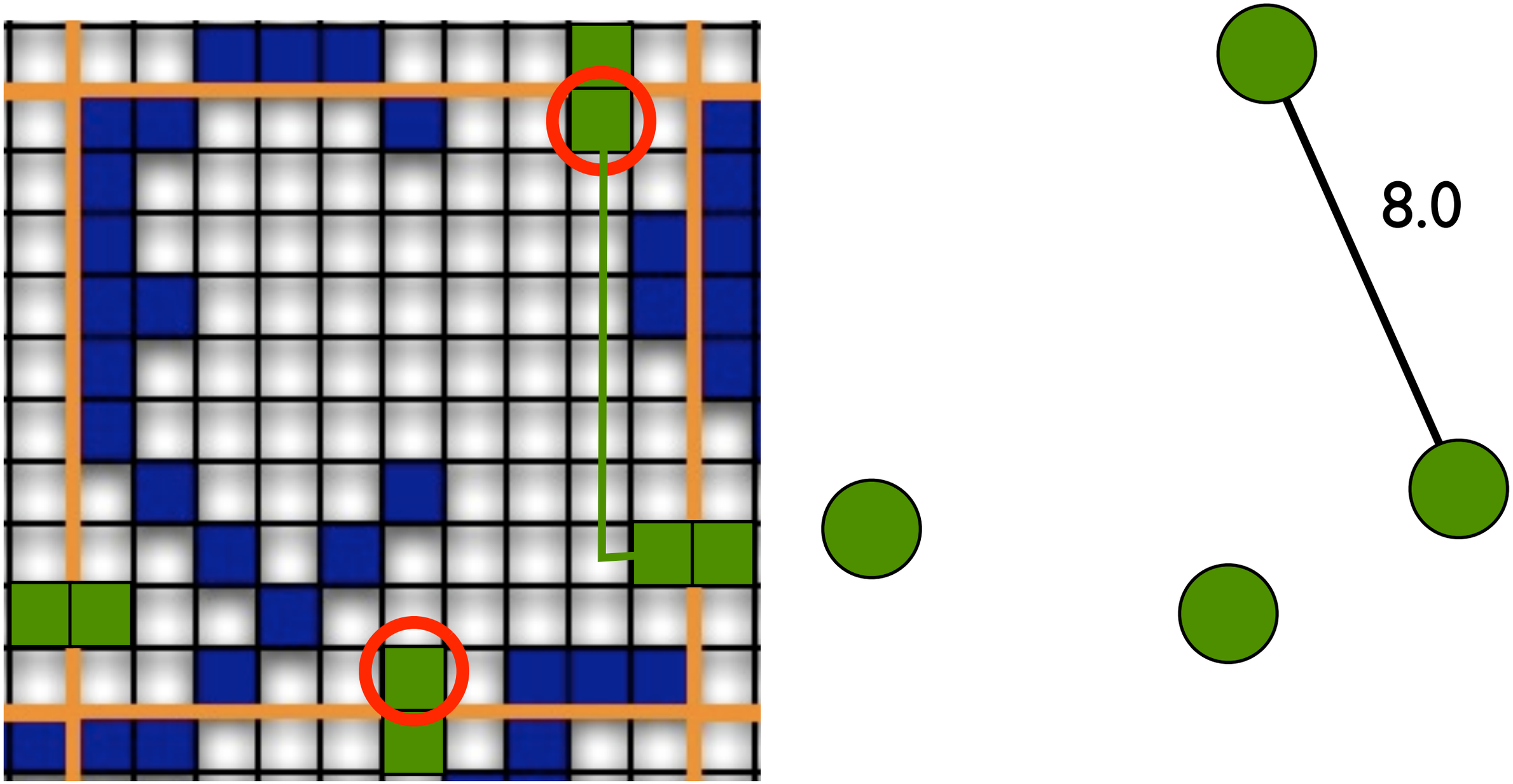


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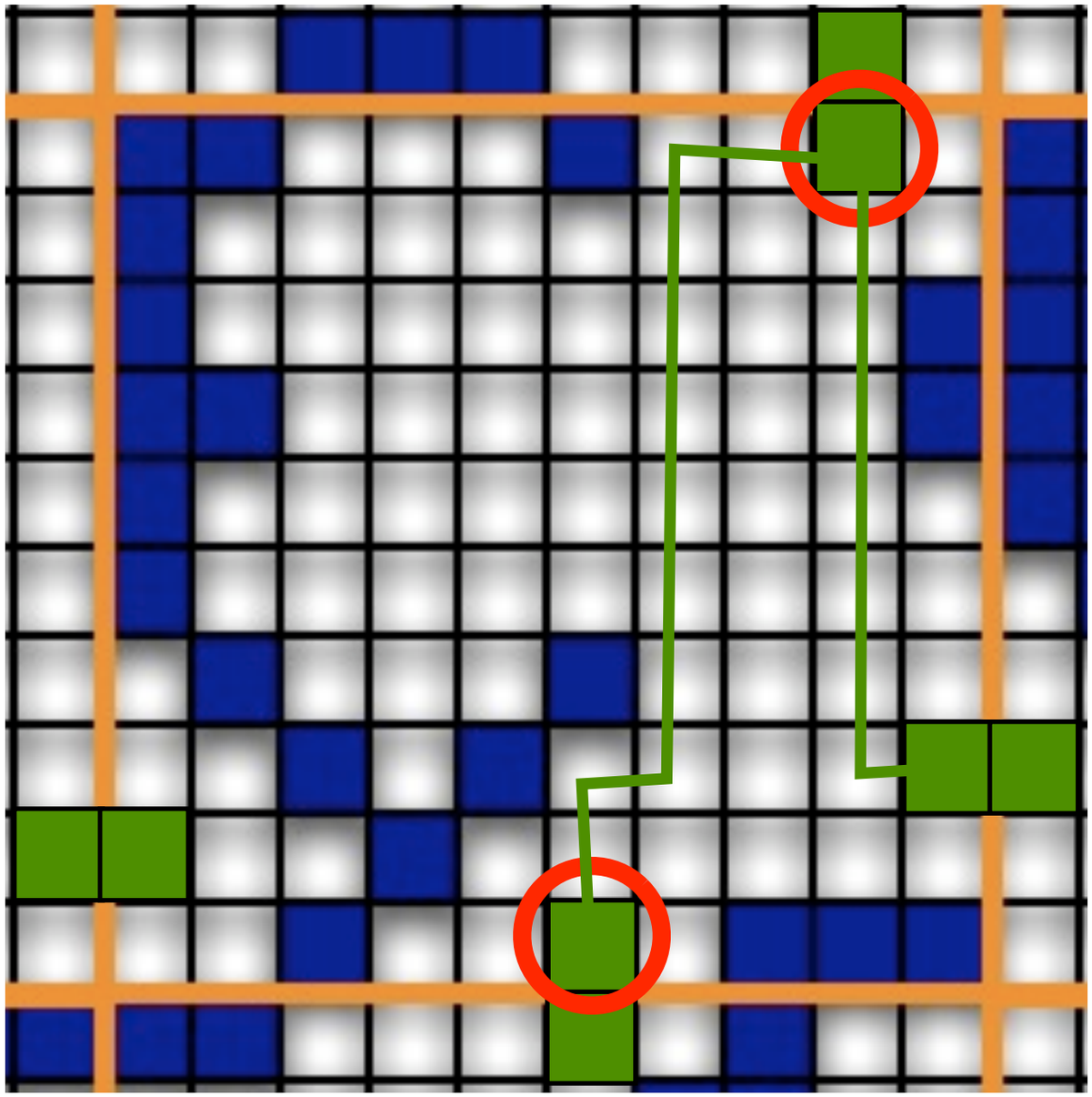

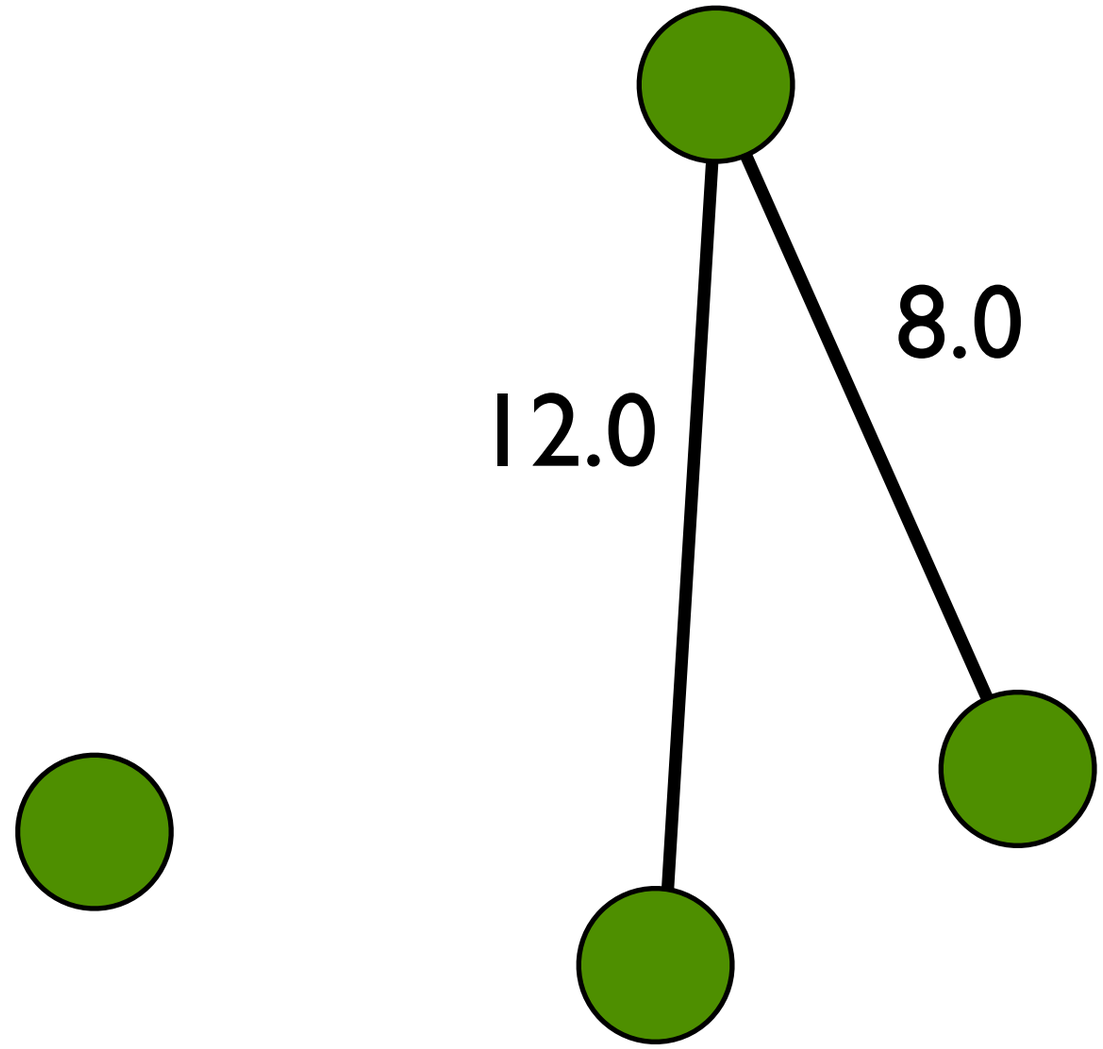


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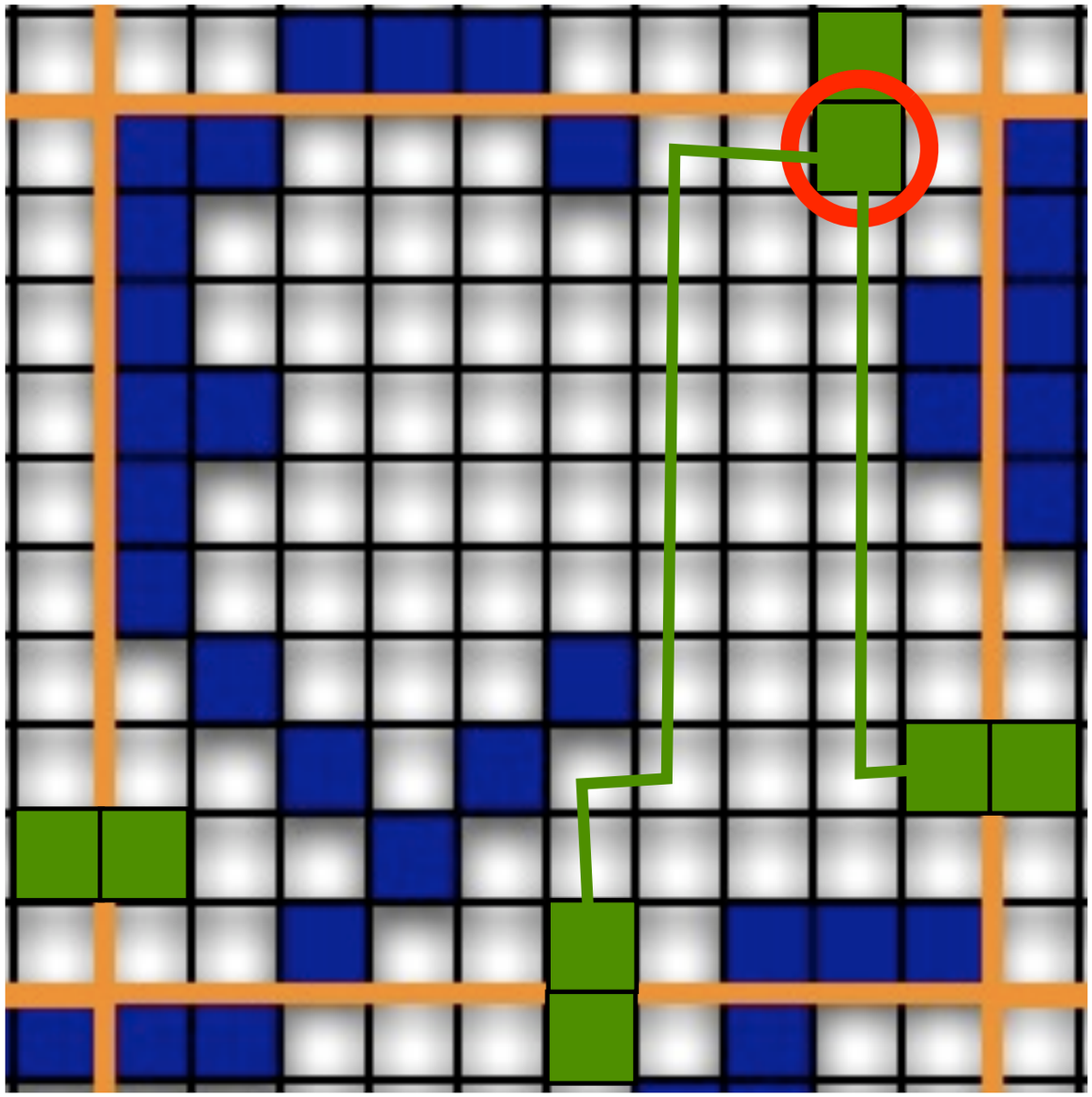

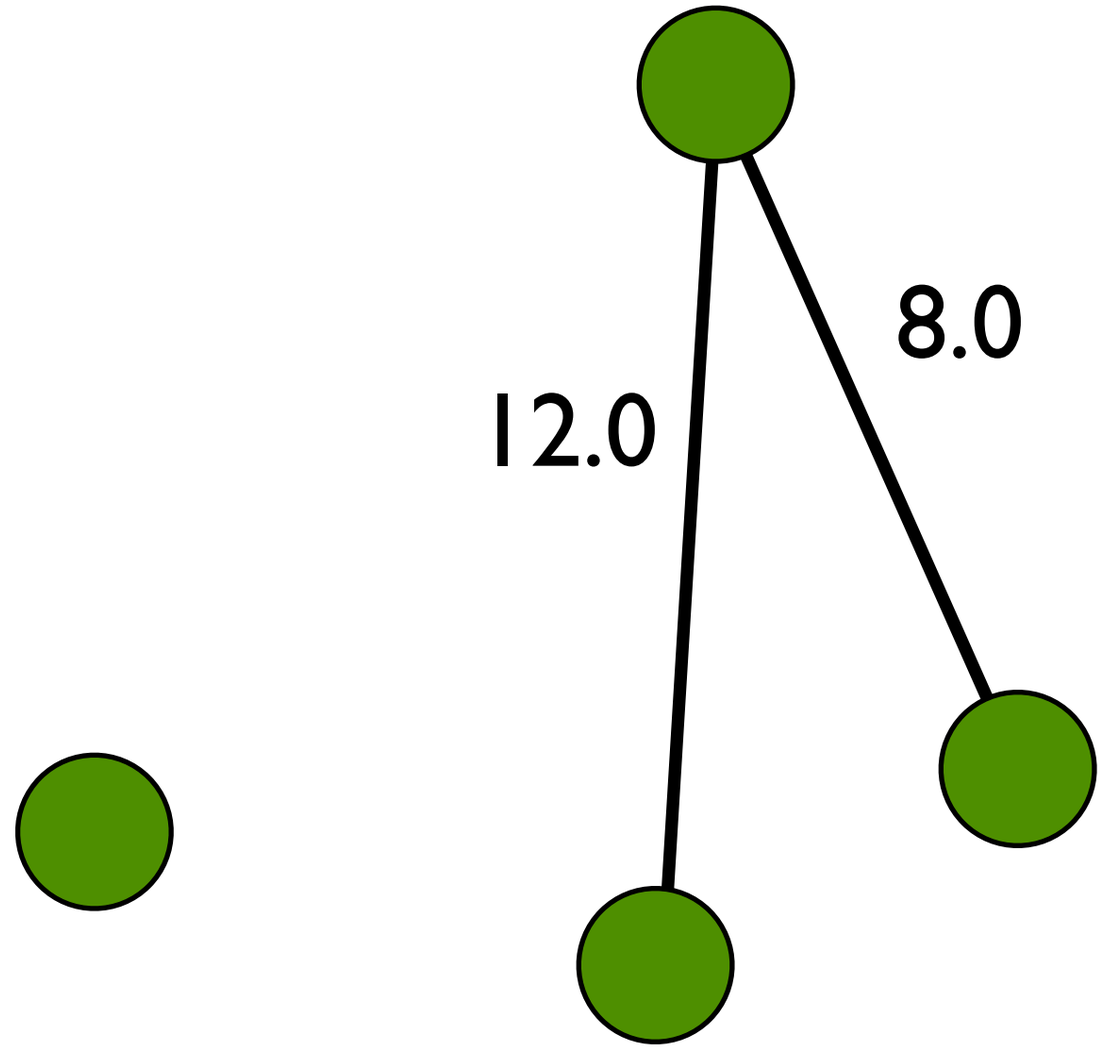


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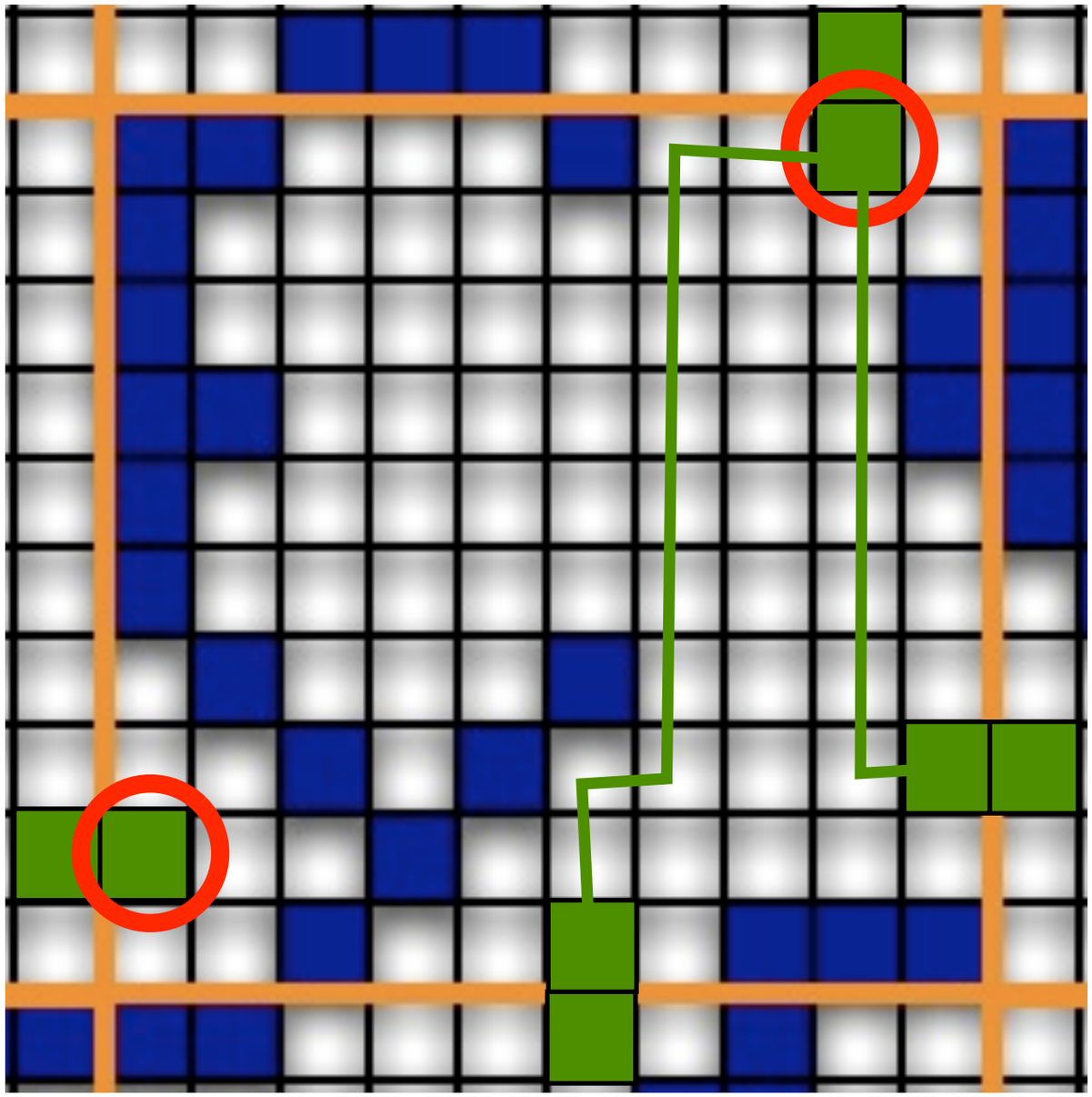

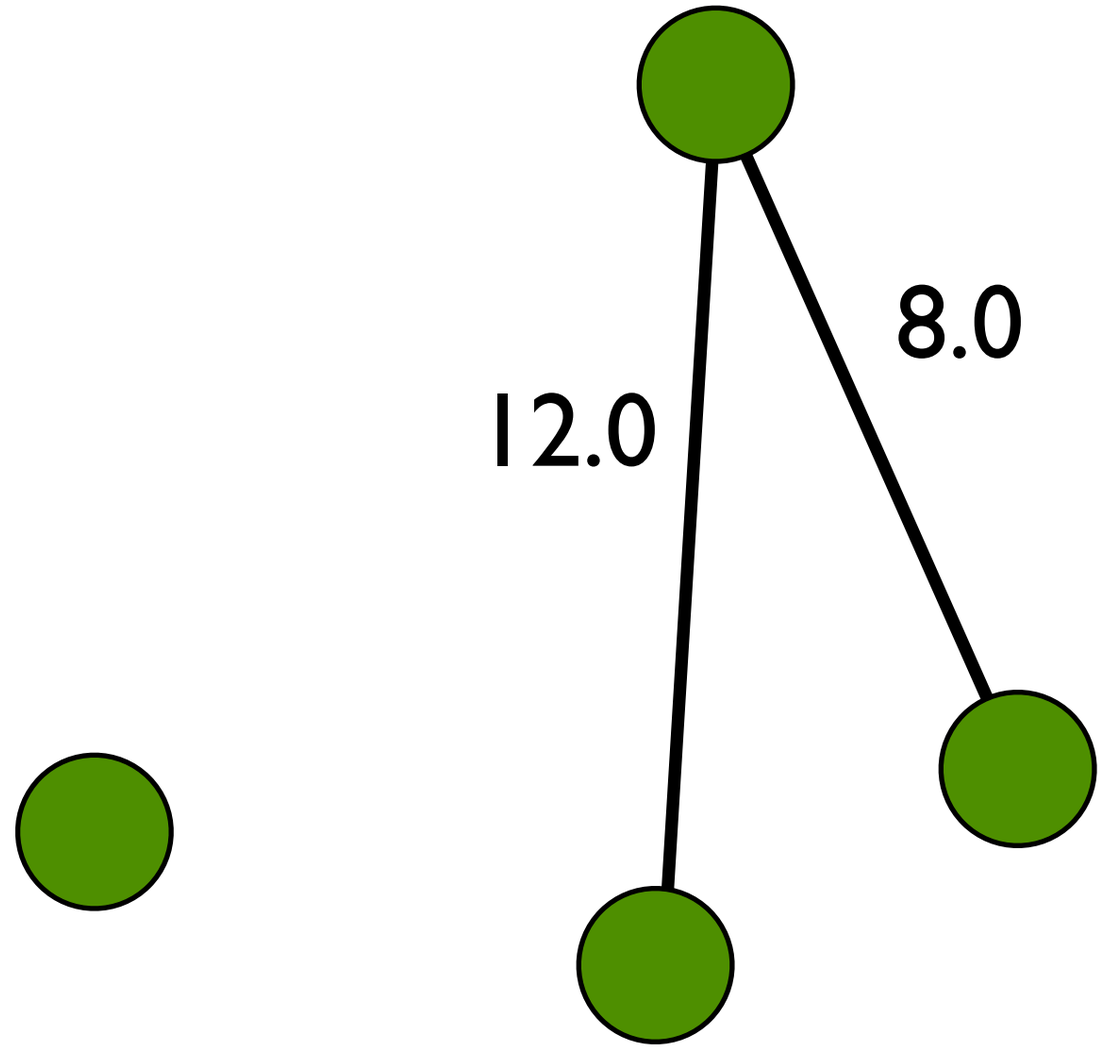


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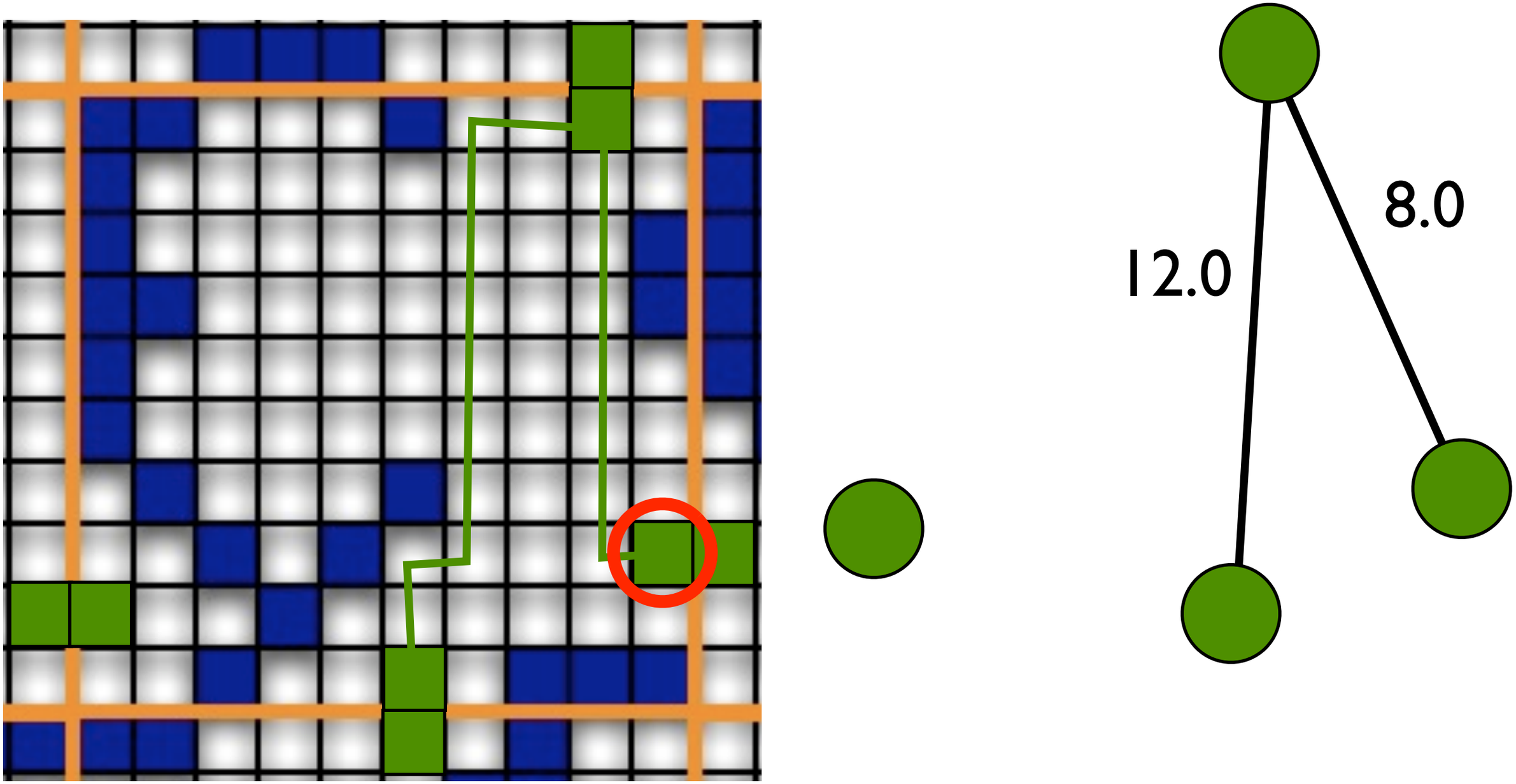
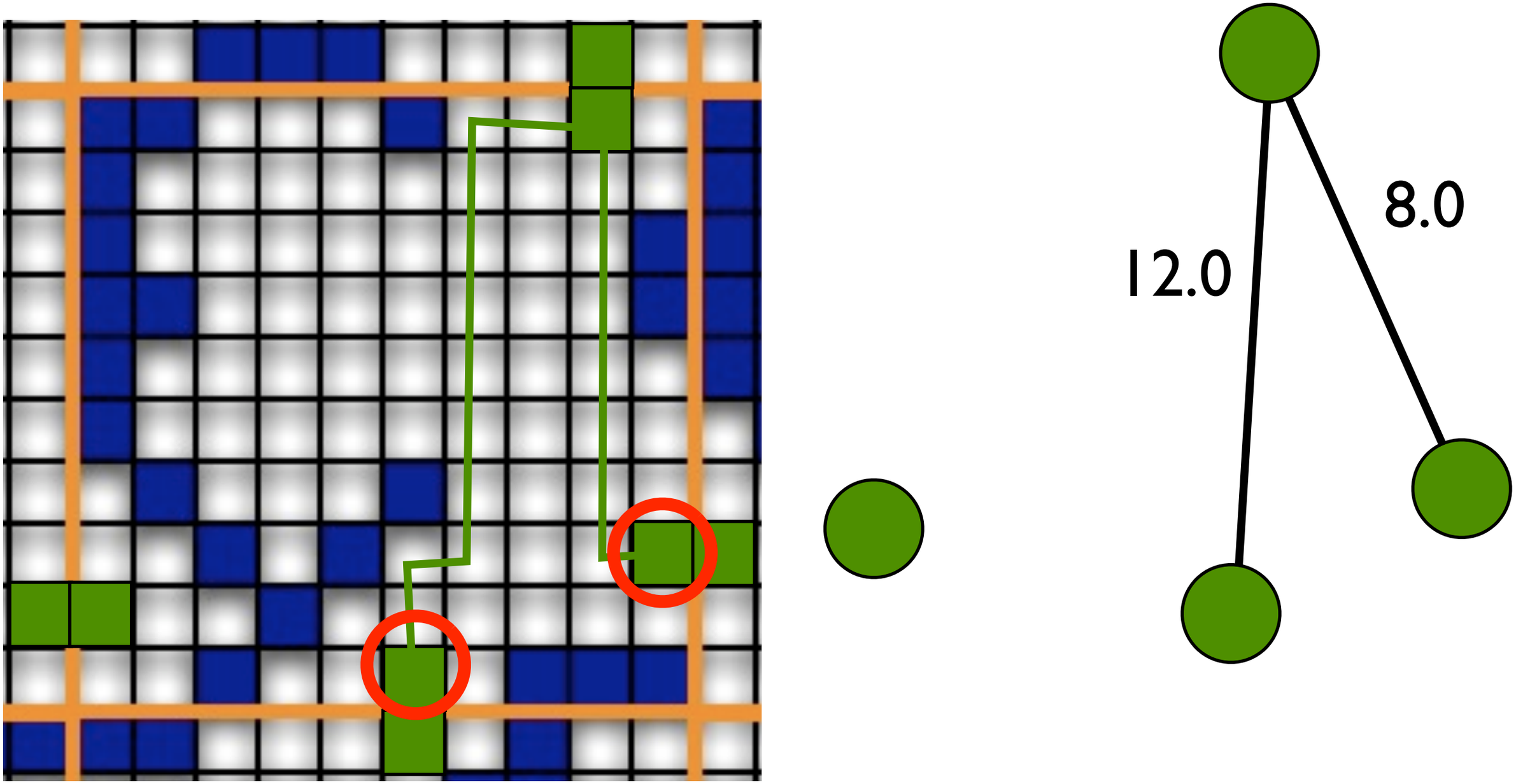



Insert intra-edges (of cost = path length) to abstract graph.

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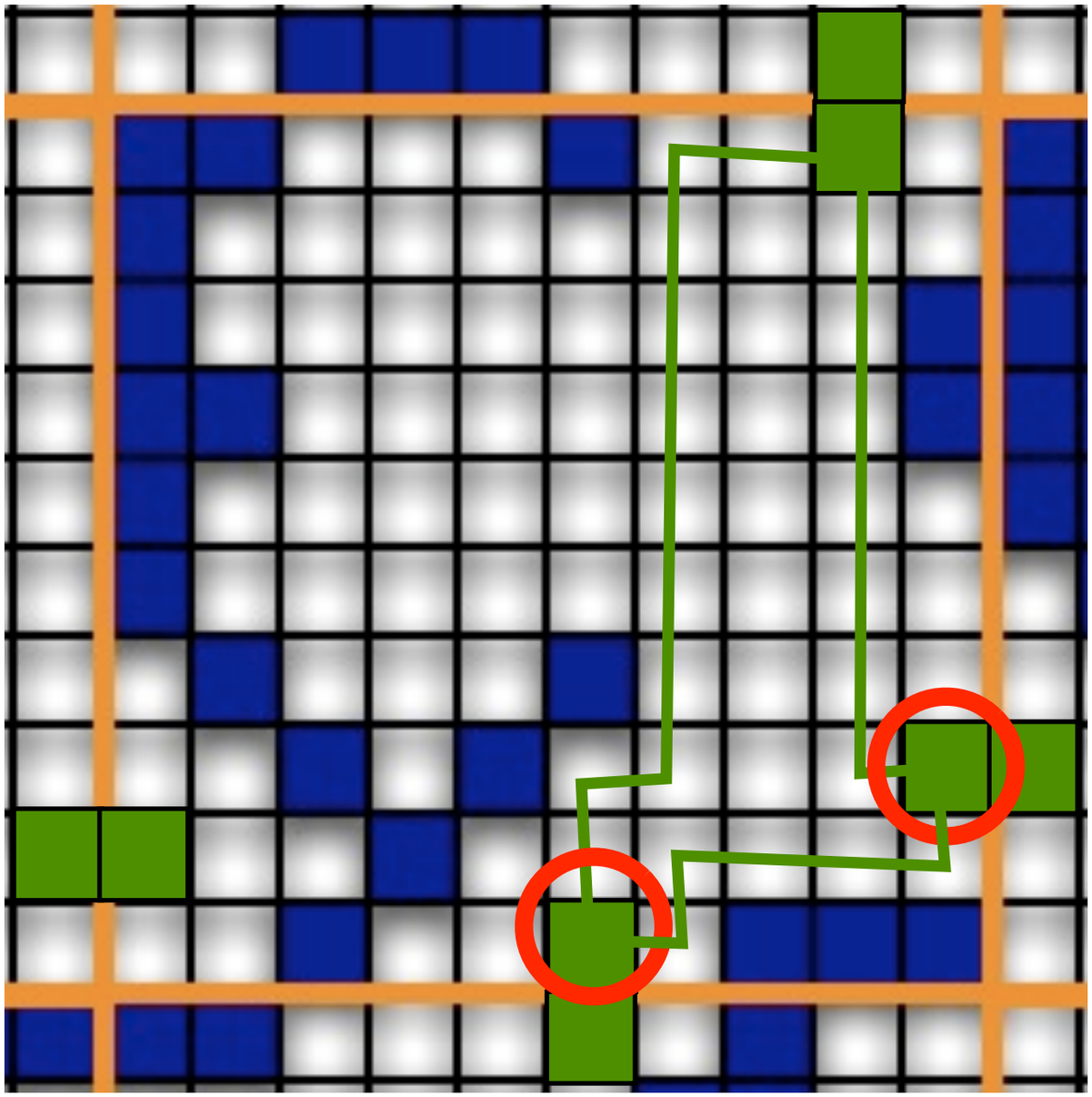

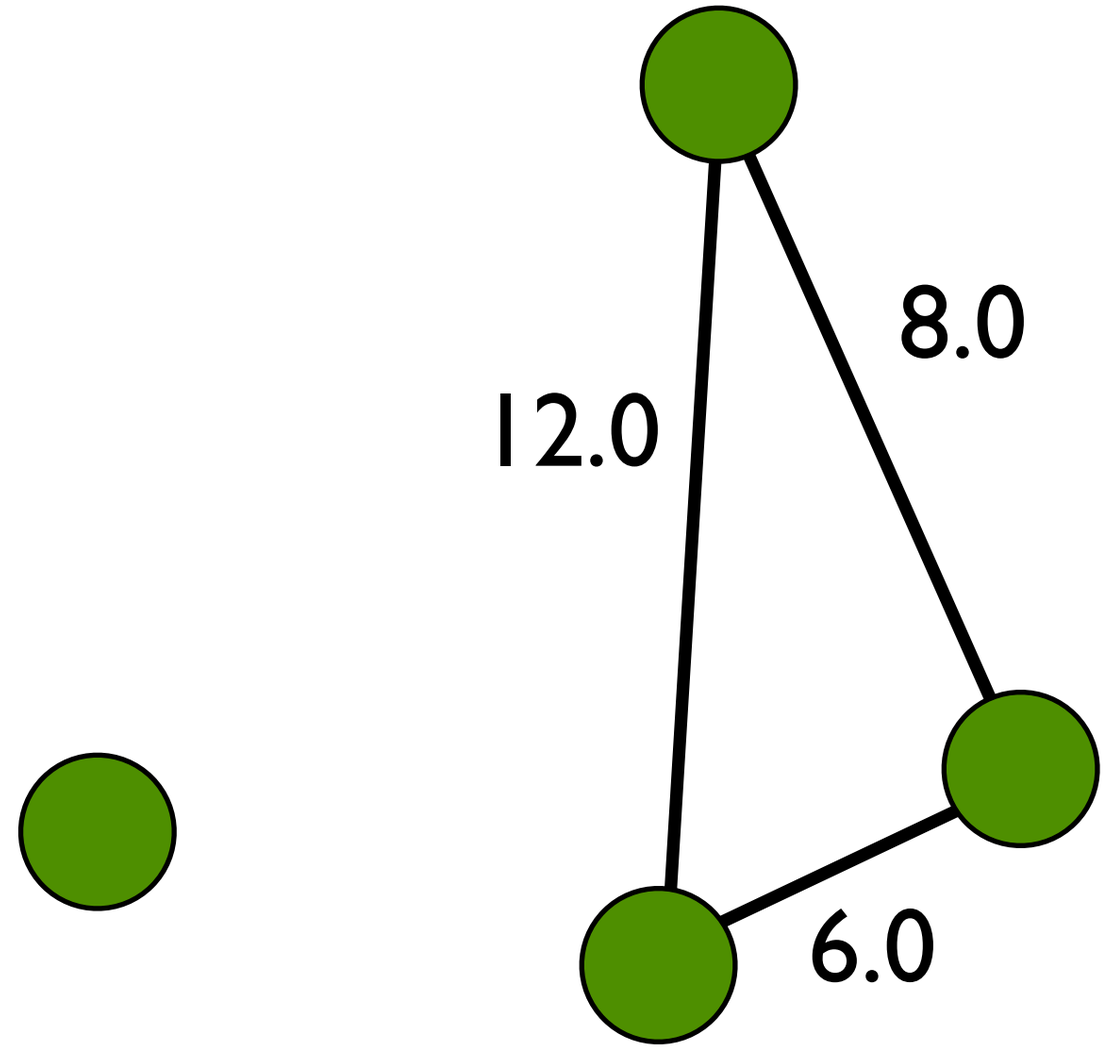


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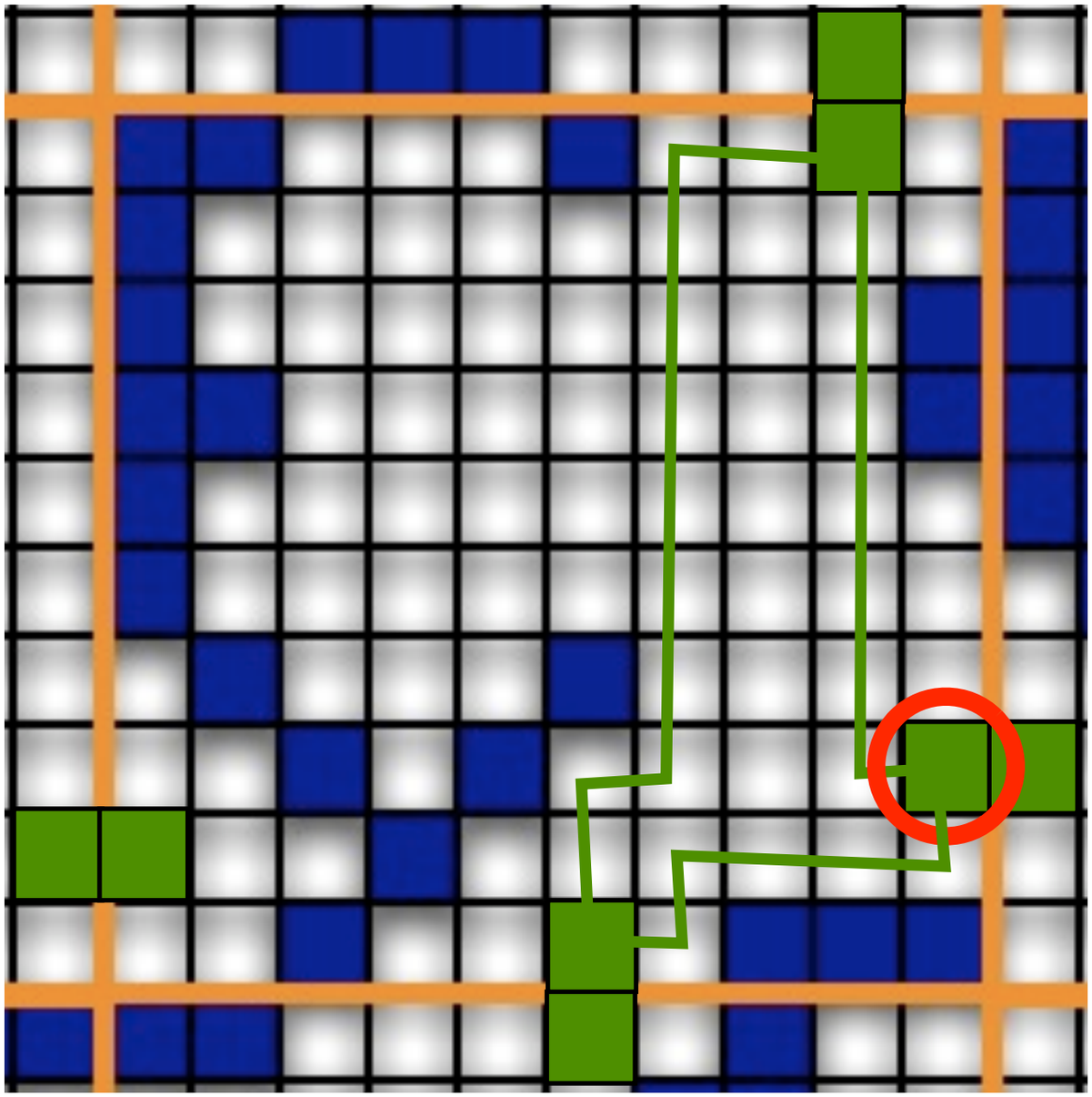

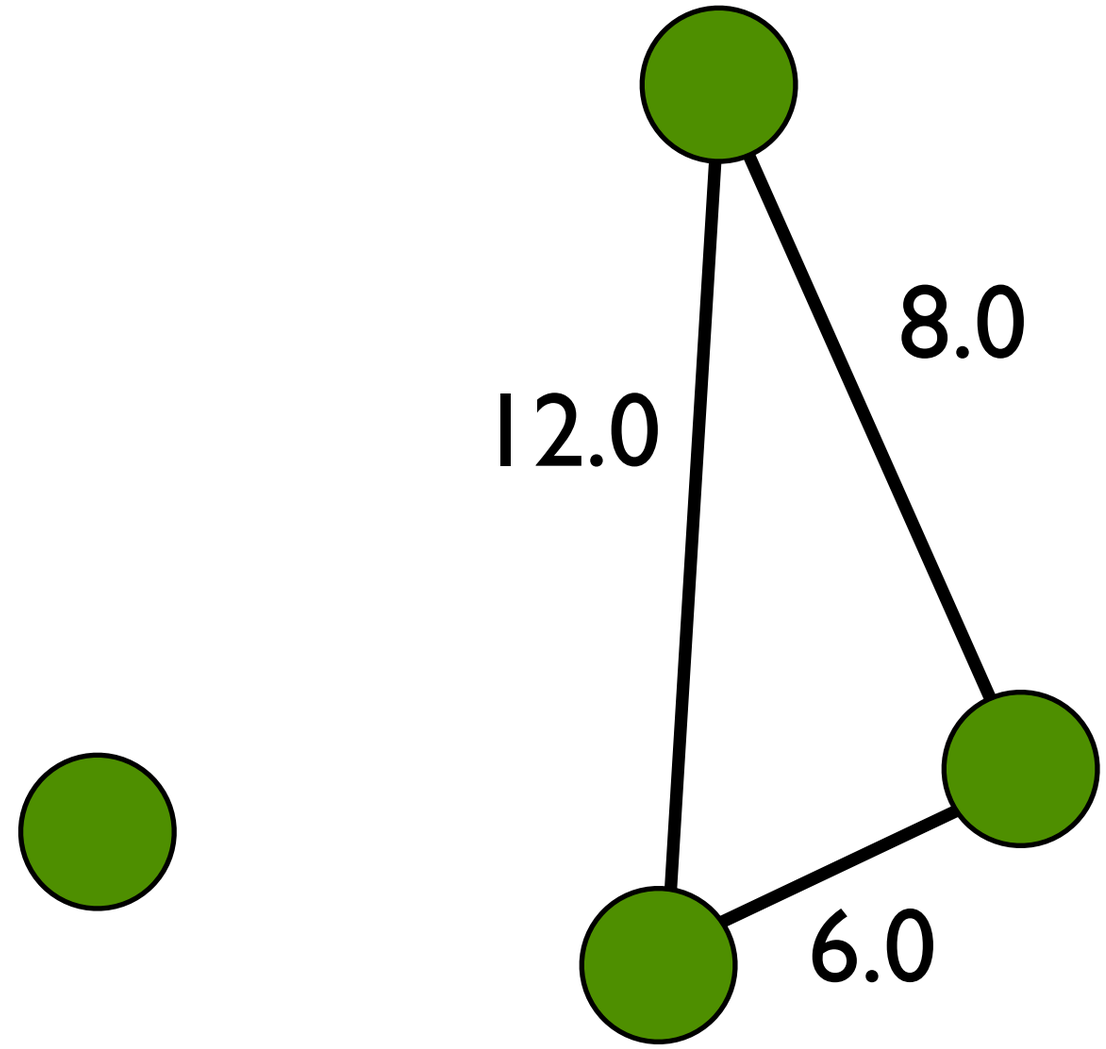


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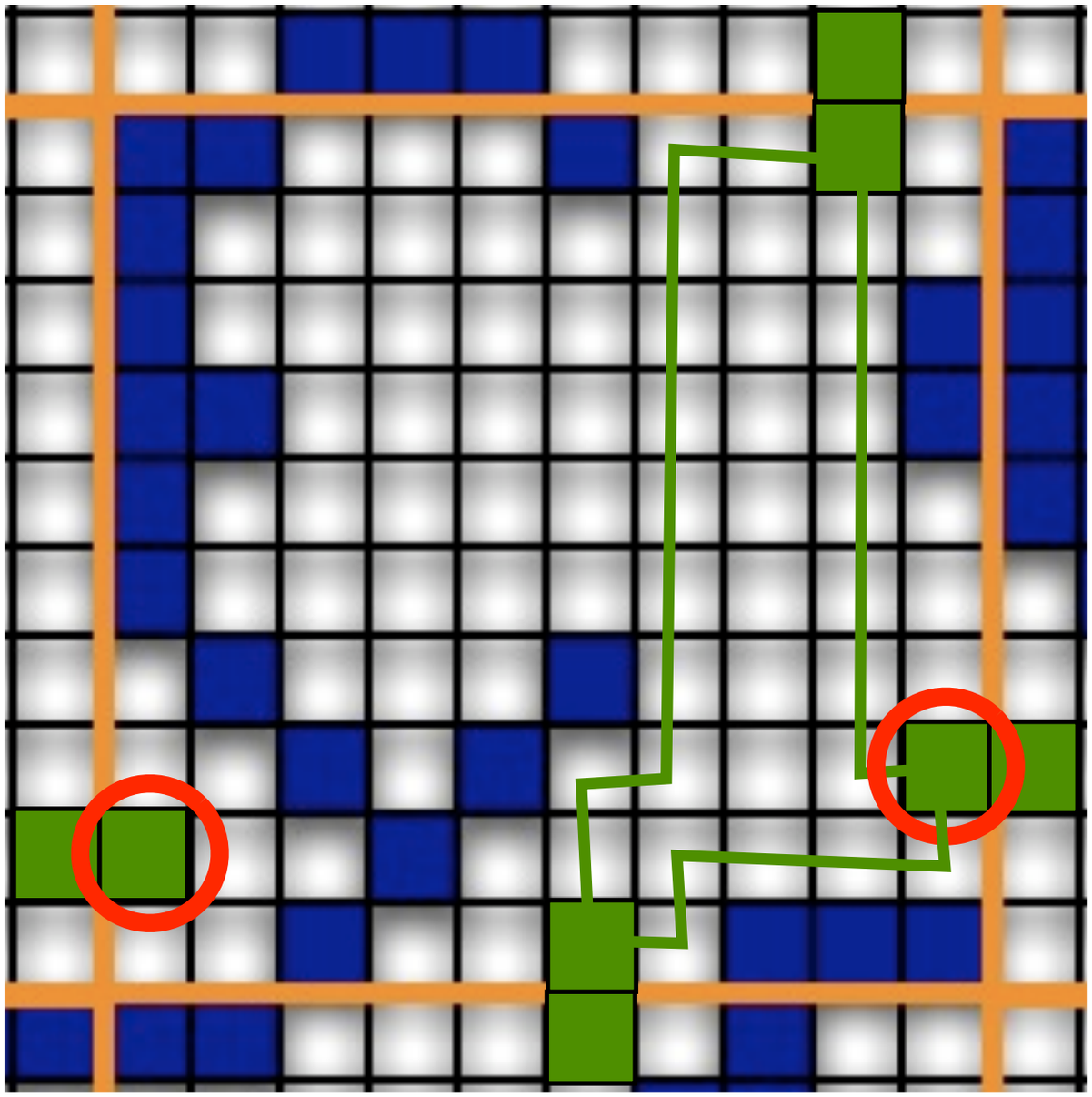

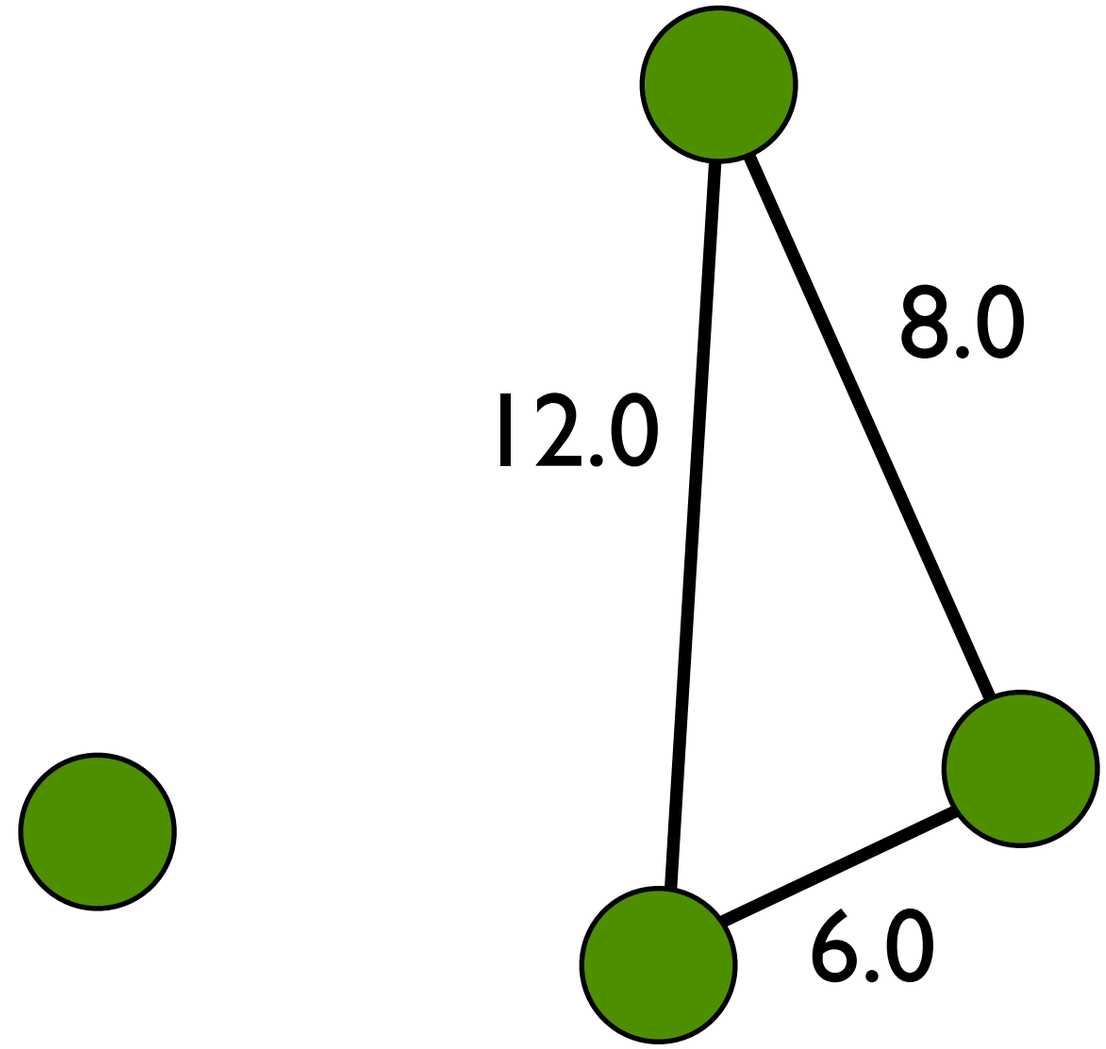


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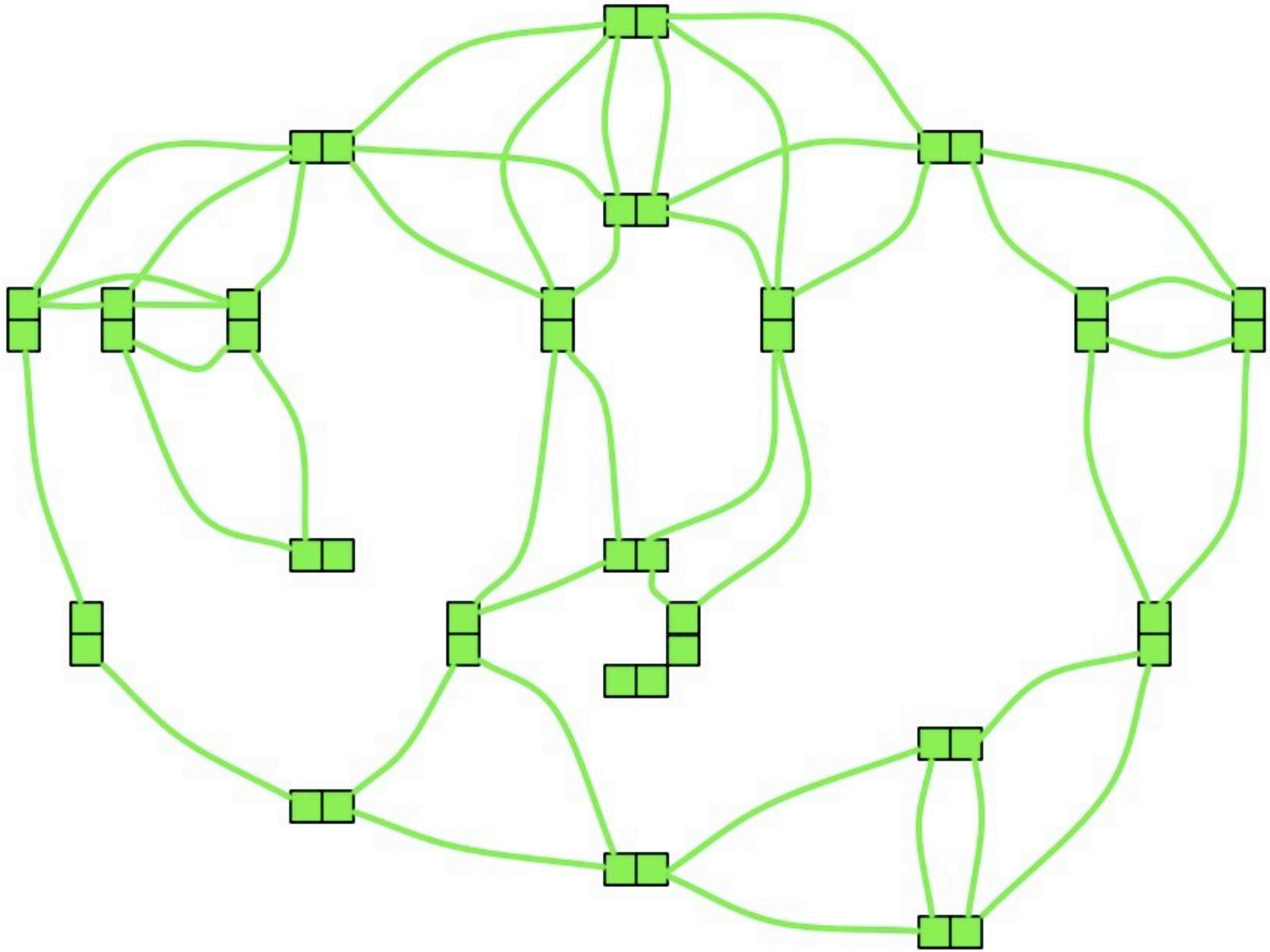
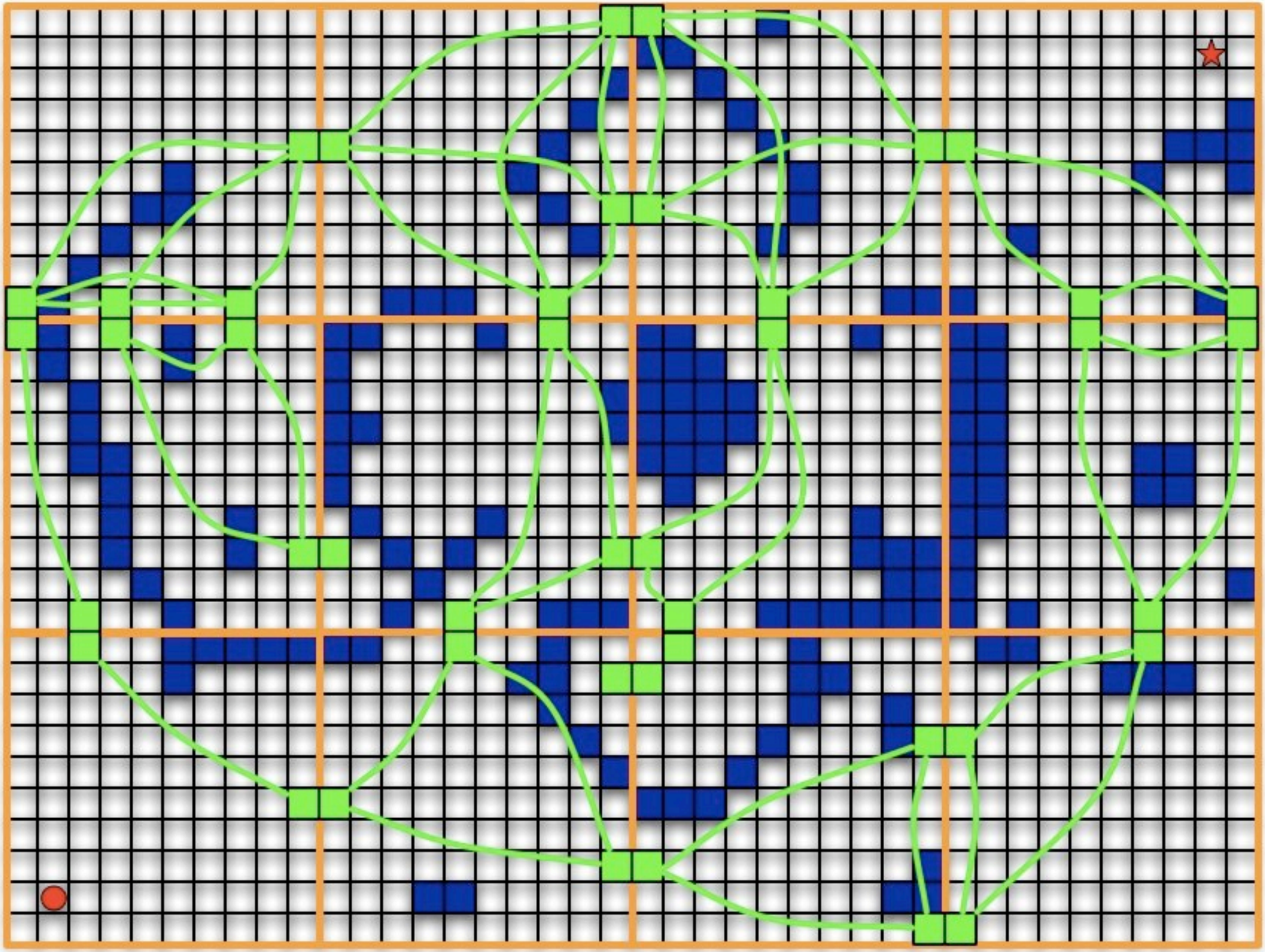


# Final abstract graph

Image: Adi Botea

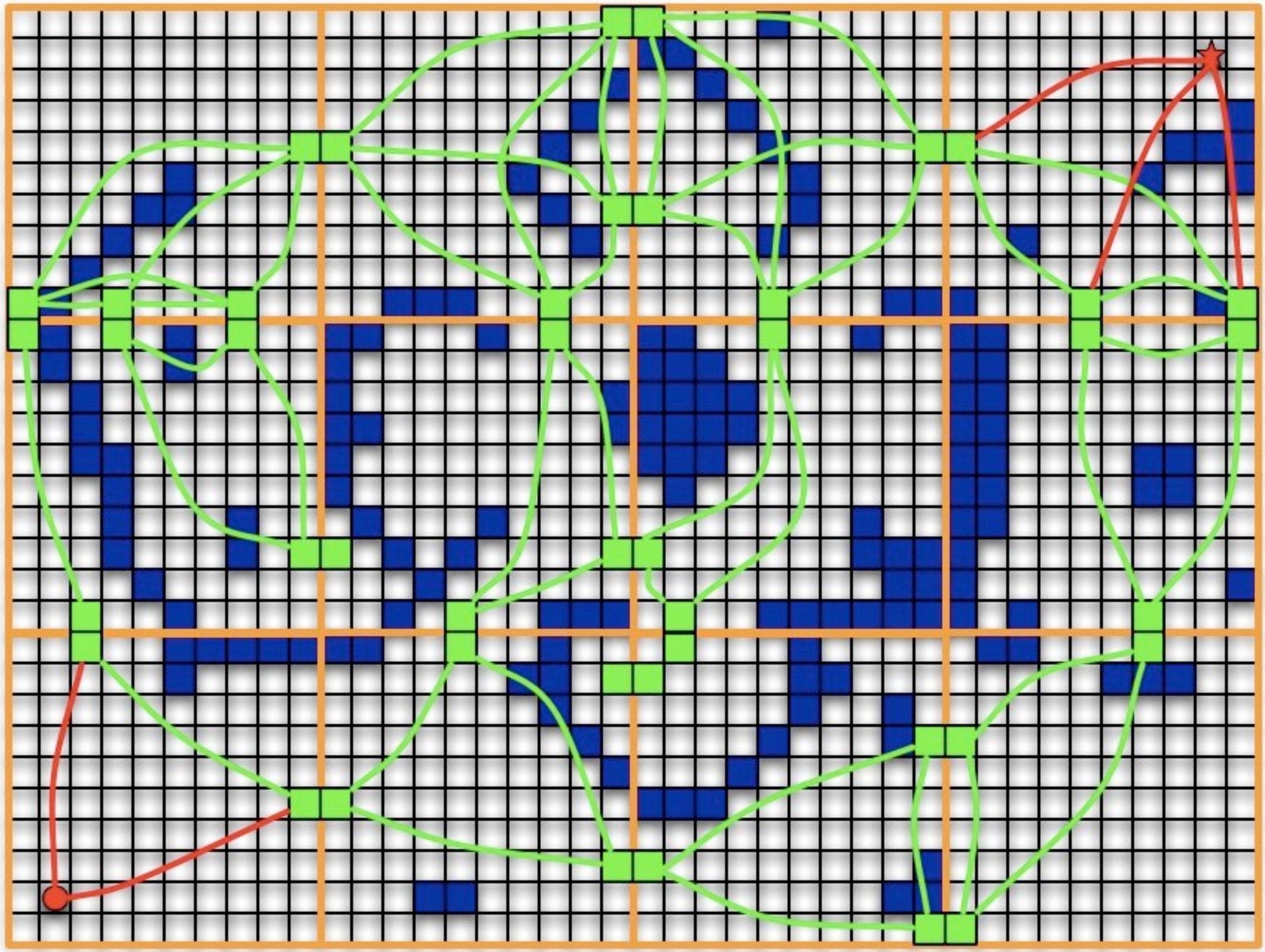
# Runtime processing

- Insert start + goal into abstract graph.
- Find an abstract path connecting them.
- Refine each abstract step using a small A* search to traverse each cluster (free if we cache intra-edge paths).



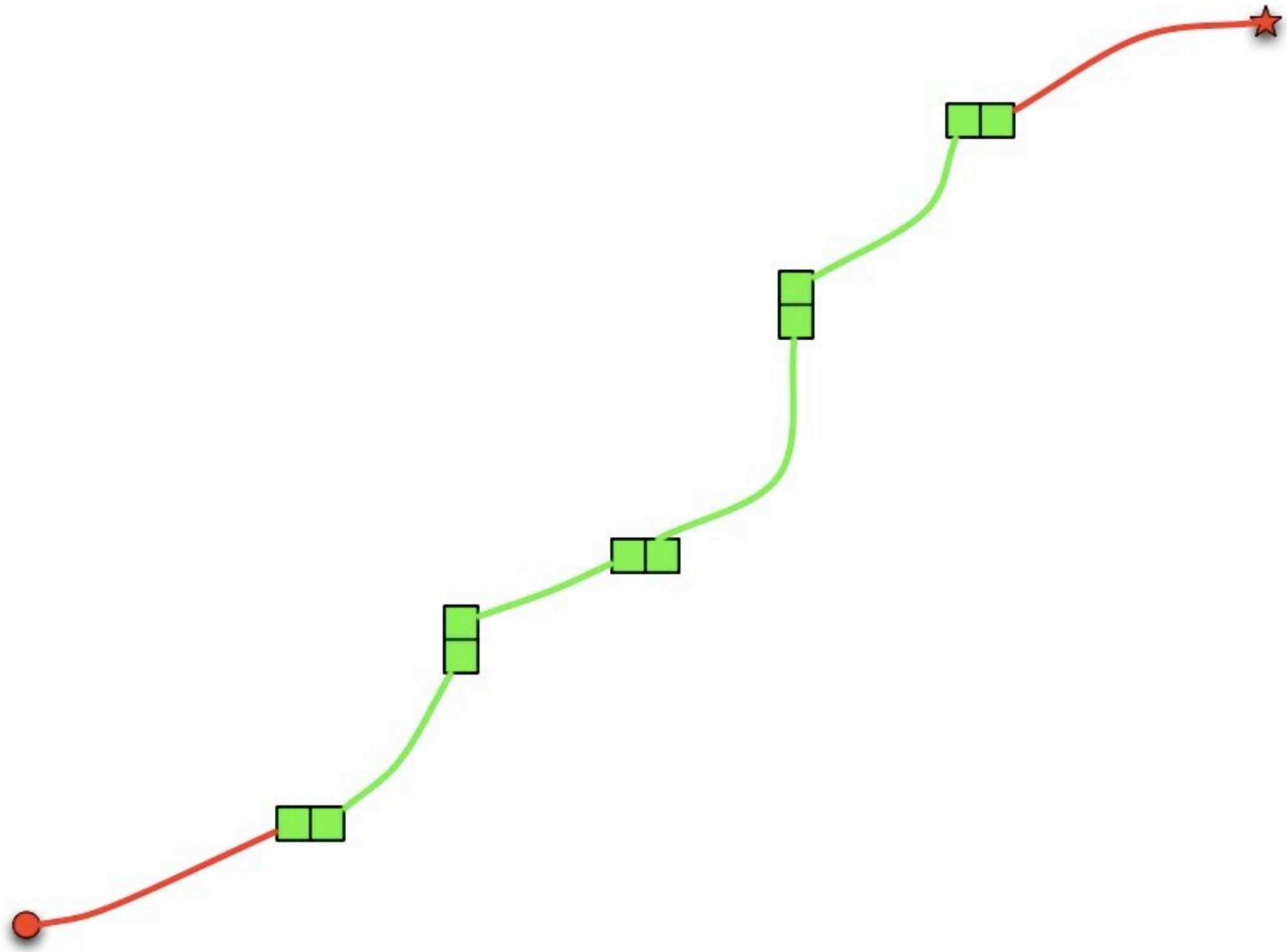
# Add Start + Goal as (temp.) abstract nodes

Image: Adi Botea



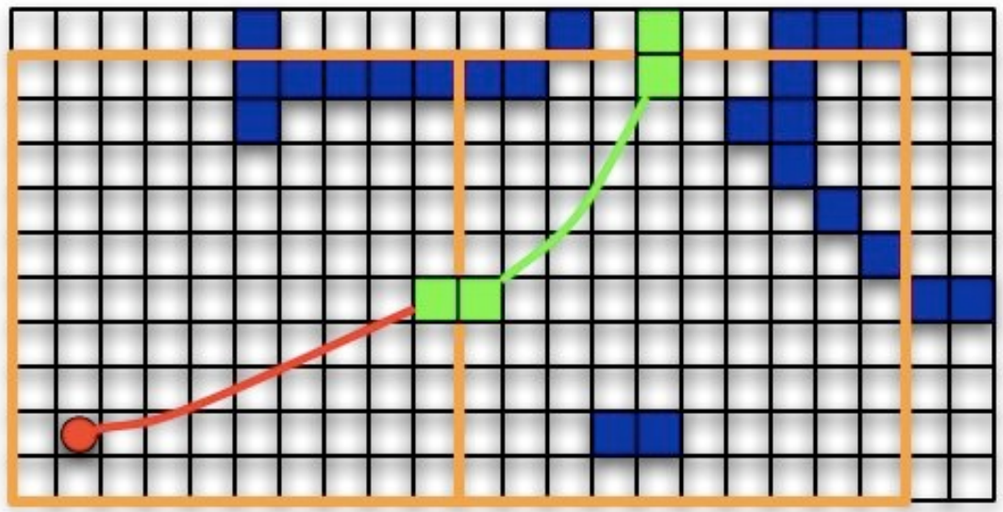
Insert temp. edges to connect Start + Goal

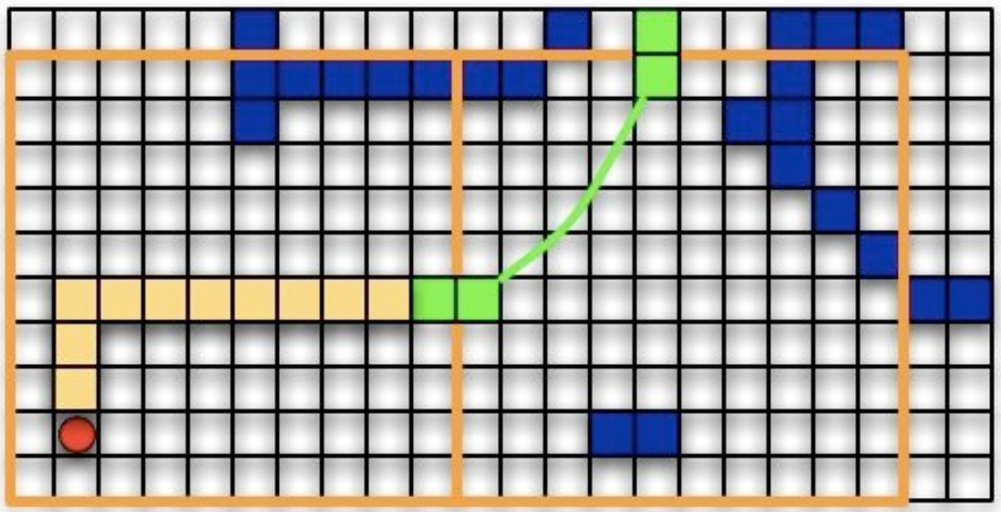
Image: Adi Botea

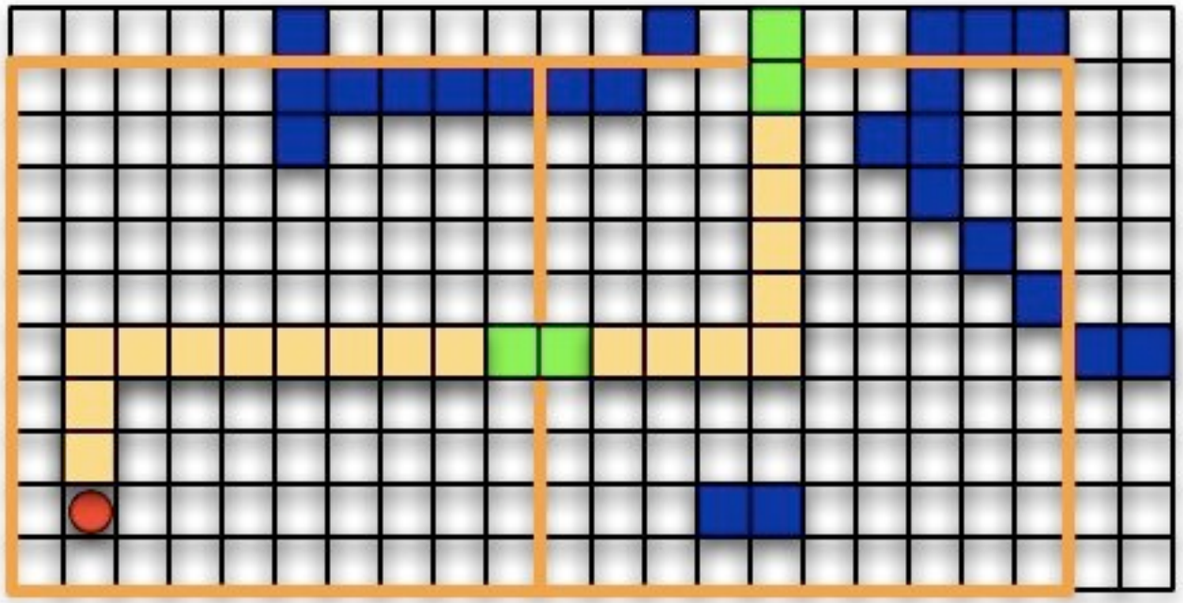


Run A* to find an abstract path

Image: Adi Botea







Refine the abstract path before execution. Each step is independent (and fast!)

Images: Adi Botea

# HPA* Advantages

- Easy to understand + implement
- Very fast (Beats A* by 10x)
- Low memory overhead (vs. A*)
- Complete

# HPA* Disadvantages

- Non-optimal (but very close!).
- Assumes fixed-size 1x1 units.
- Only supports single terrain type.
- Insertion effort.

# Part 2:
# Dealing with diversity

# Assumptions in pathfinding literature

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- Assumption #1: all units are the same size.

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- Assumption #2: any terrain traversable for one unit is traversable for all.

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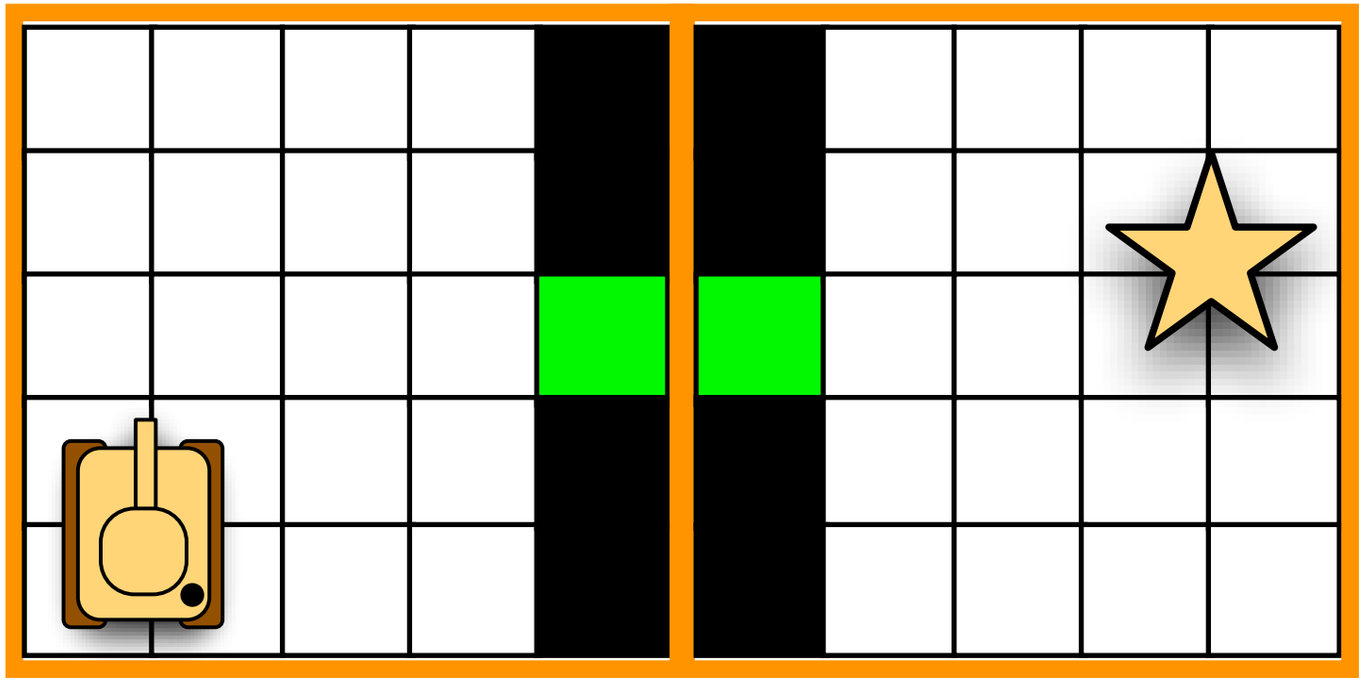
- Assumption #1: all units are the same size.
- Assumption #2: any terrain traversable for one unit is traversable for all.
- Pathfinding breaks when either assumption is lifted.

# HPA* Tank Fail



HPA* loses information (like sizes of entrances)

# Challenging pathfinding

Unit sizes: Small, Medium, Large
Terrains: Snow, Road, Water, Ground

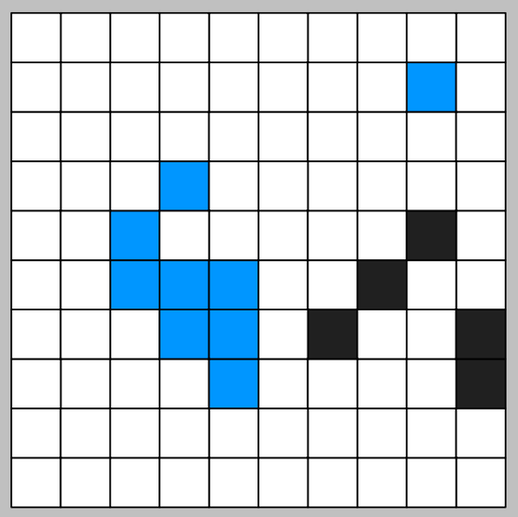
Images: Red Alert 3 (Electronic Arts)

# HAA* Overview

- [Harabor, Botea, 2008]
- HPA*-style abstraction & search.
- Combined with clearance-based pathfinding.
- Caters for different terrains and unit sizes.
- Constructs complete abstractions.

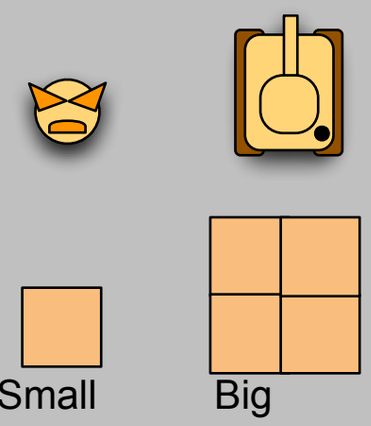
# Modeling diversity

## Example Map

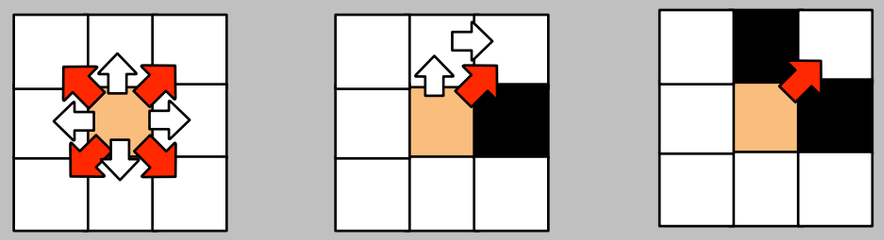


Ground (white tiles)
Water (blue tiles)
Obstacles (black tiles)

## Agent Sizes




## Movement rules




## Terrain traversal Capabilities

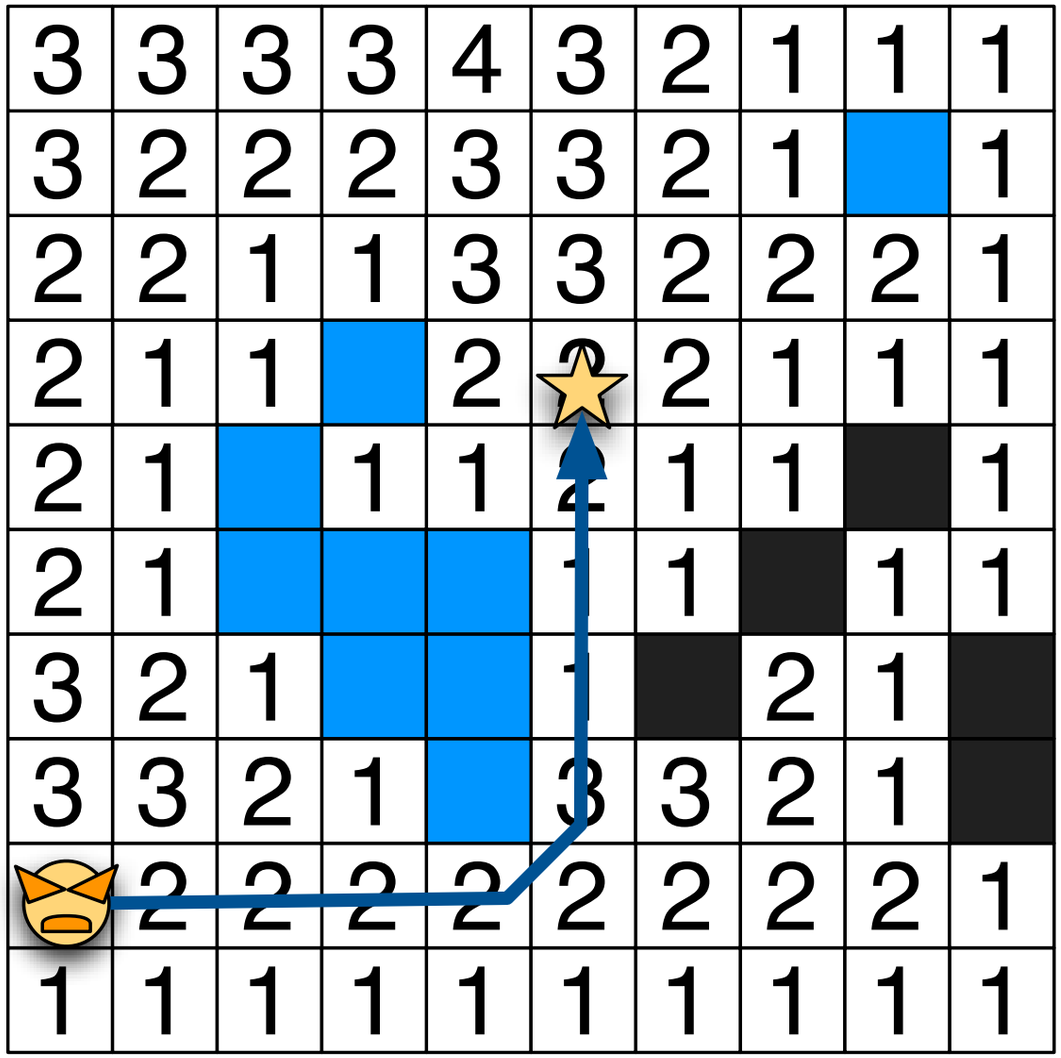
{Ground}
{Water}
{Ground or Water}

# Clearance-based pathfinding

- Intuition:
  - Calculate how much clearance (traversable space) exists at a given tile (t) on the map.
  - A tile (t) is traversable by an agent (a) if:

    1. terrain(t) is in capability(a)
    2. clearance(t, capability(a)) >= size(a)

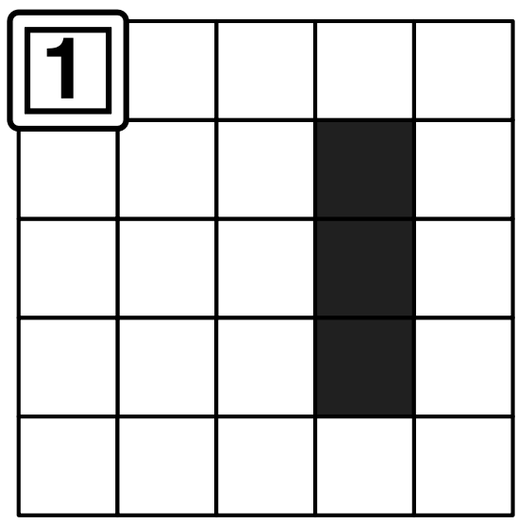
# Example




Agent size = 1
Capability = {Ground}

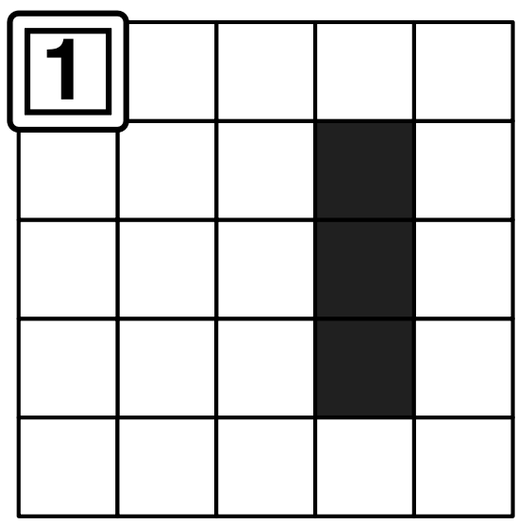
Each tile on the path is traversable for the agent

# Computing clearance

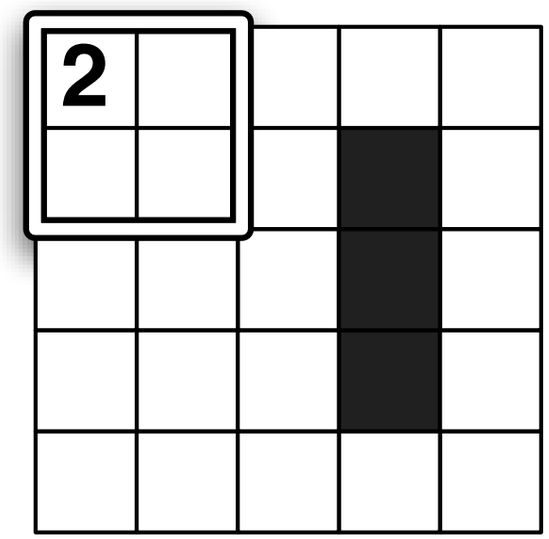



Initial Clearance.
Capability = {Ground}
(white tiles)

# Computing clearance

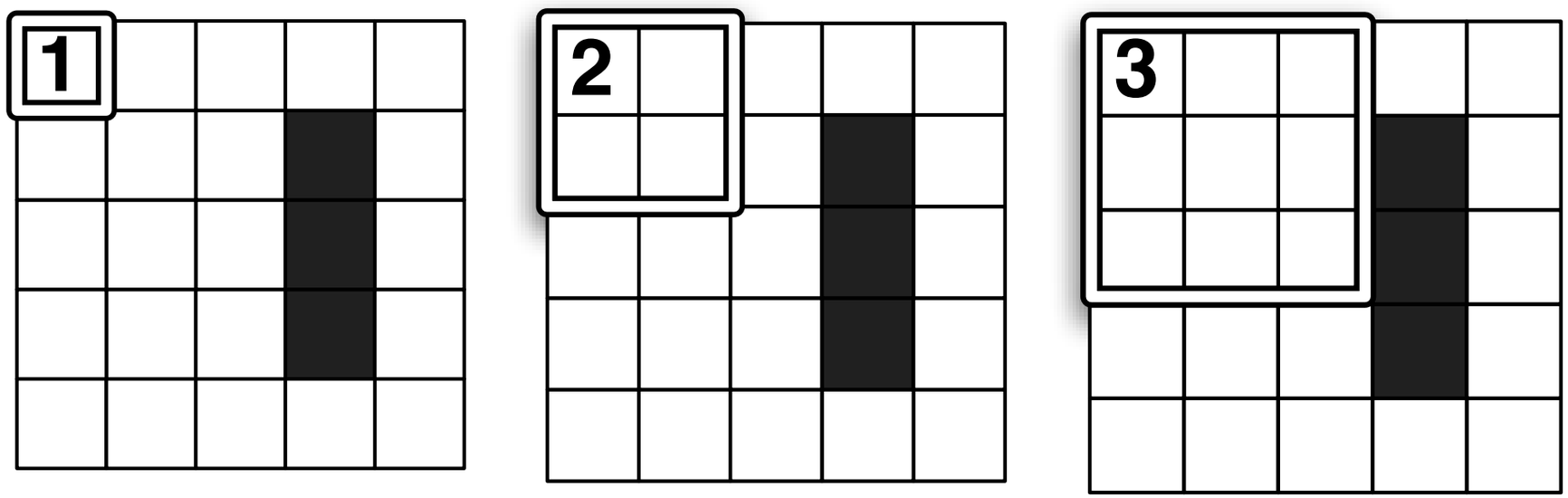



Initial Clearance.
Capability = {Ground}
(white tiles)




First Expansion
OK

# Computing clearance

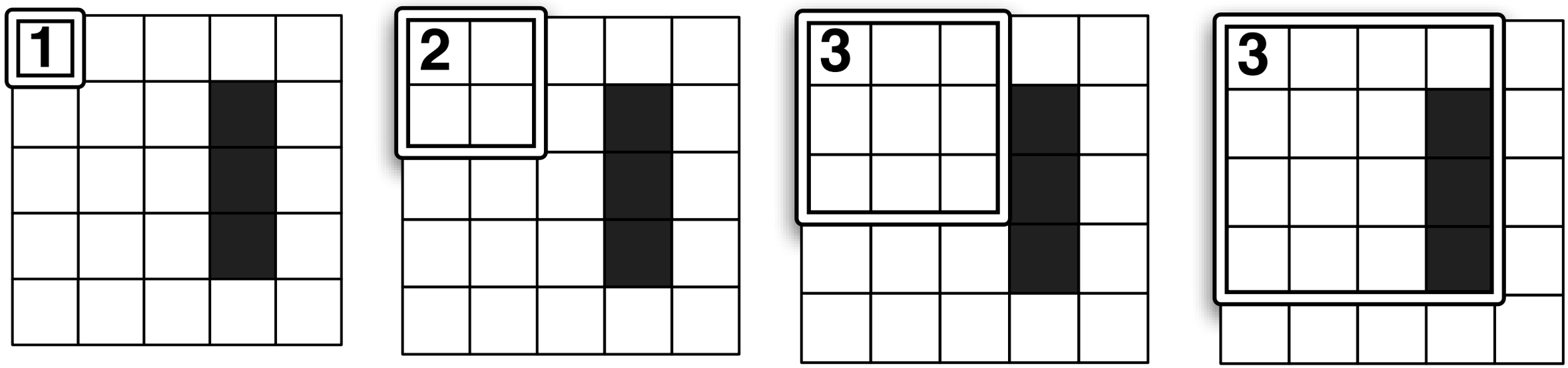



Initial Clearance.
Capability = {Ground}
(white tiles)

First Expansion
OK

Second Expansion
OK

# Computing clearance




Initial Clearance.
Capability = {Ground}
(white tiles)

First Expansion
OK

Second Expansion
OK

Third Expansion
Fail

# Maps with multiple terrain types

- Easy! Each tile has a clearance-value for each possible capability.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 3 | 3 | 3 | 4 | 3 | 2 | 1 | 1 | 1 |
| 3 | 2 | 2 | 2 | 3 | 3 | 2 | 1 | | 1 |
| 2 | 2 | 1 | 1 | 3 | 3 | 2 | 2 | 2 | 1 |
| 2 | 1 | 1 | | 2 | 2 | 2 | 1 | 1 | 1 |
| 2 | 1 | | 1 | 1 | 2 | 1 | 1 | | 1 |
| 2 | 1 | | | | 1 | 1 | | 1 | 1 |
| 3 | 2 | 1 | | | 1 | | 2 | 1 | |
| 3 | 3 | 2 | 1 | | 3 | 3 | 2 | 1 | |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Clearances for {Ground} Capability

# Maps with multiple terrain types

- Easy! Each tile has a clearance-value for each possible capability.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 3 | 3 | 3 | 4 | 3 | 2 | 1 | 1 | 1 |
| 3 | 2 | 2 | 2 | 3 | 3 | 2 | 1 | | 1 |
| 2 | 2 | 1 | 1 | 3 | 3 | 2 | 2 | 2 | 1 |
| 2 | 1 | 1 | | 2 | 2 | 2 | 1 | 1 | 1 |
| 2 | 1 | | 1 | 1 | 2 | 1 | 1 | | 1 |
| 2 | 1 | | | | 1 | 1 | | 1 | 1 |
| 3 | 2 | 1 | | | 1 | | 2 | 1 | |
| 3 | 3 | 2 | 1 | | 3 | 3 | 2 | 1 | |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Clearances for {Ground} Capability

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | | | | | | | | 1 | |
| | | | | | | | | | |
| | | | 1 | | | | | | |
| | | 1 | | | | | | | |
| | | 1 | 2 | 1 | | | | | |
| | | | 1 | 1 | | | | | |
| | | | | 1 | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

Clearances for {Water} Capability

# Maps with multiple terrain types

- Easy! Each tile has a clearance-value for each possible capability.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 3 | 3 | 3 | 4 | 3 | 2 | 1 | 1 | 1 |
| 3 | 2 | 2 | 2 | 3 | 3 | 2 | 1 | | 1 |
| 2 | 2 | 1 | 1 | 3 | 3 | 2 | 2 | 2 | 1 |
| 2 | 1 | 1 | | 2 | 2 | 2 | 1 | 1 | 1 |
| 2 | 1 | | 1 | 1 | 2 | 1 | 1 | | 1 |
| 2 | 1 | | | | 1 | 1 | | 1 | 1 |
| 3 | 2 | 1 | | | 1 | | 2 | 1 | |
| 3 | 3 | 2 | 1 | | 3 | 3 | 2 | 1 | |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Clearances for {Ground} Capability

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | | | | | | | | 1 | |
| | | | | | | | | | |
| | | | 1 | | | | | | |
| | | 1 | | | | | | | |
| | | 1 | 2 | 1 | | | | | |
| | | | 1 | 1 | | | | | |
| | | | | 1 | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

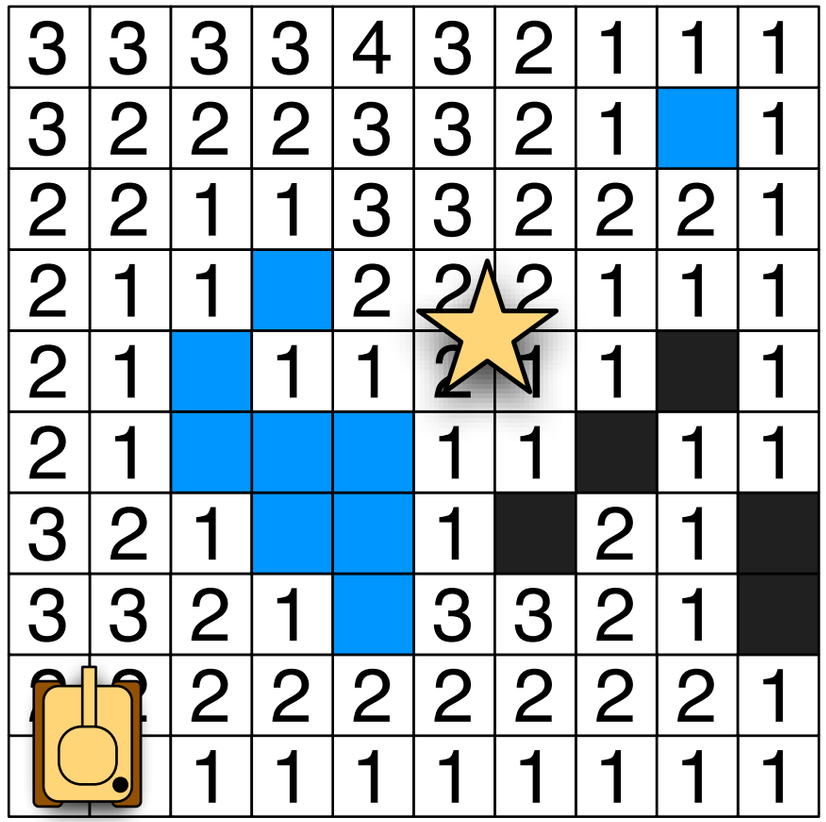
Clearances for {Water} Capability

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 6 | 5 | 5 | 4 | 4 | 4 | 3 | 2 | 1 |
| 6 | 5 | 5 | 4 | 4 | 3 | 3 | 3 | 2 | 1 |
| 6 | 5 | 4 | 4 | 3 | 3 | 2 | 2 | 2 | 1 |
| 6 | 5 | 4 | 3 | 3 | 2 | 2 | 1 | 1 | 1 |
| 6 | 5 | 4 | 3 | 2 | 2 | 1 | 1 | | 1 |
| 5 | 5 | 4 | 3 | 2 | 1 | 1 | | 1 | 1 |
| 4 | 4 | 4 | 3 | 2 | 1 | | 2 | 1 | |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 1 | |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Clearances for {Ground, Water} Capability

# Dealing with large agents

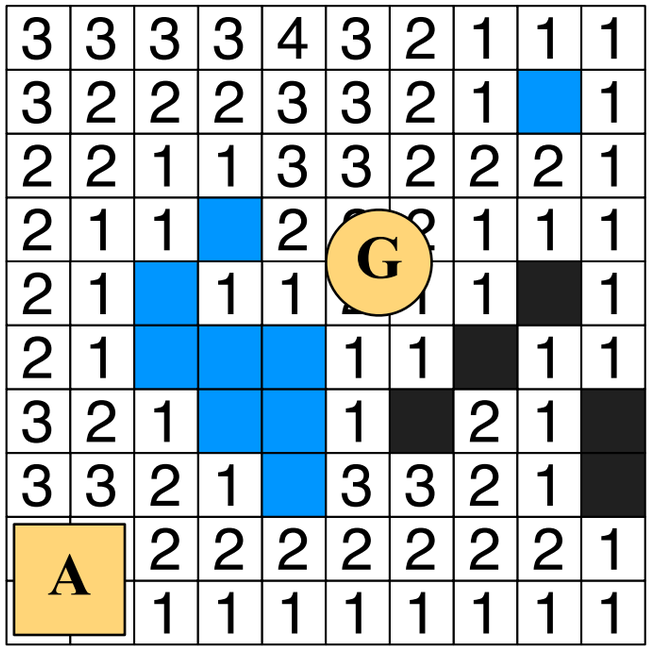
Each 2x2 agent occupies 4 tiles.
Which clearance value do we consider?




Agent size = 2, Capability = {Ground}

# Problem reduction

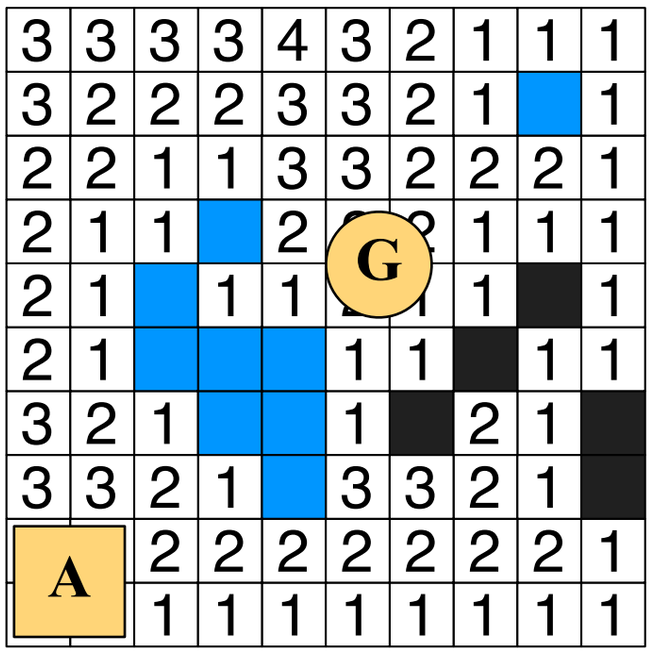
Theorem: Any pathfinding problem can be reduced into a canonical problem (agent size = 1, capability = 1 terrain)



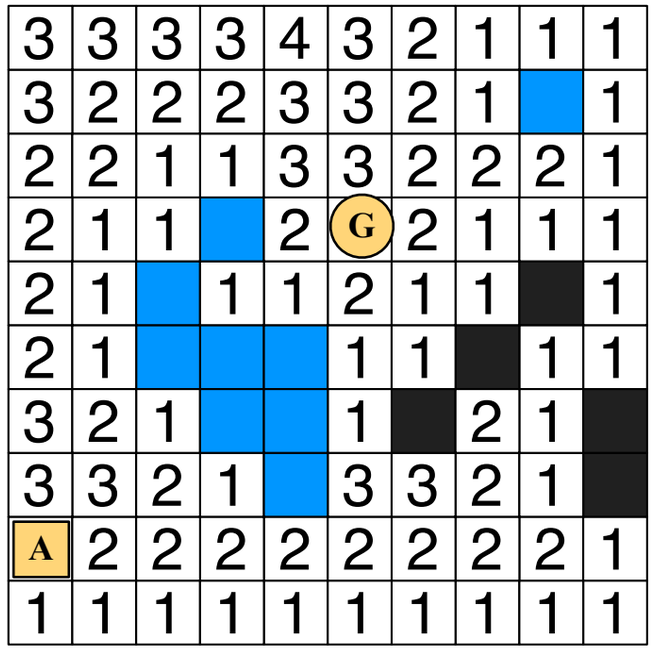

Initial problem

# Problem reduction

Theorem: Any pathfinding problem can be reduced into a canonical problem (agent size = 1, capability = 1 terrain)



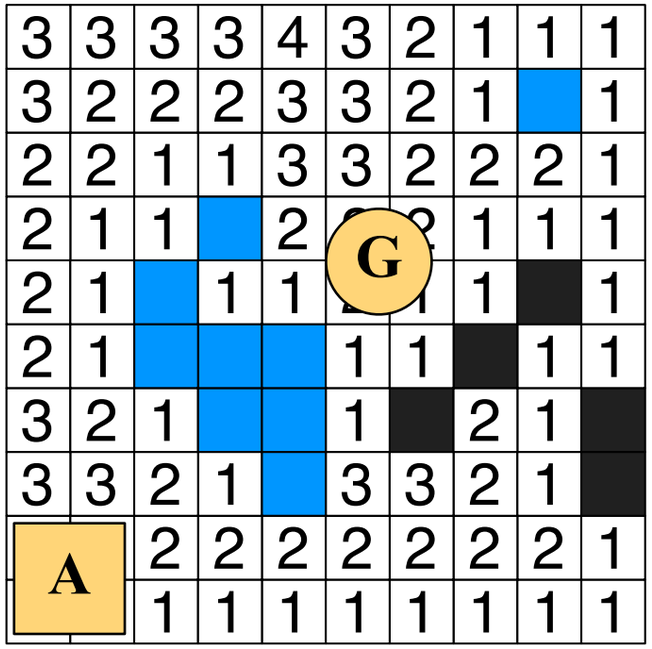

Initial problem



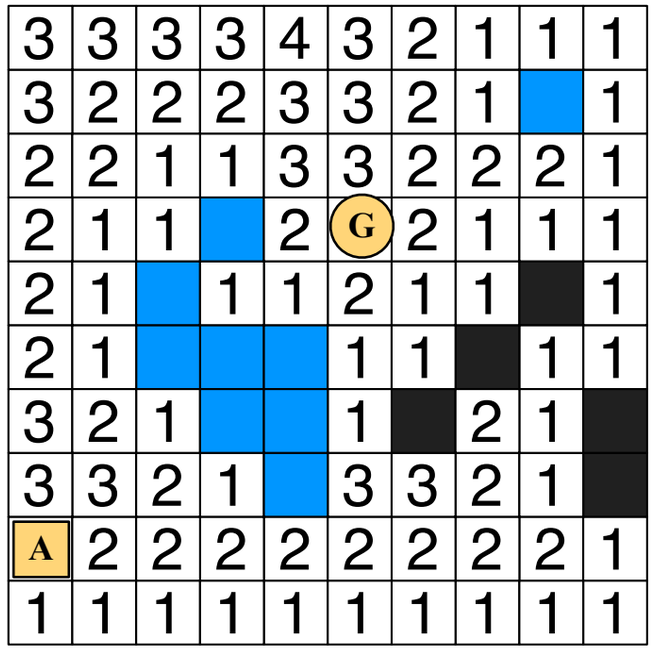

Reduced size

# Problem reduction

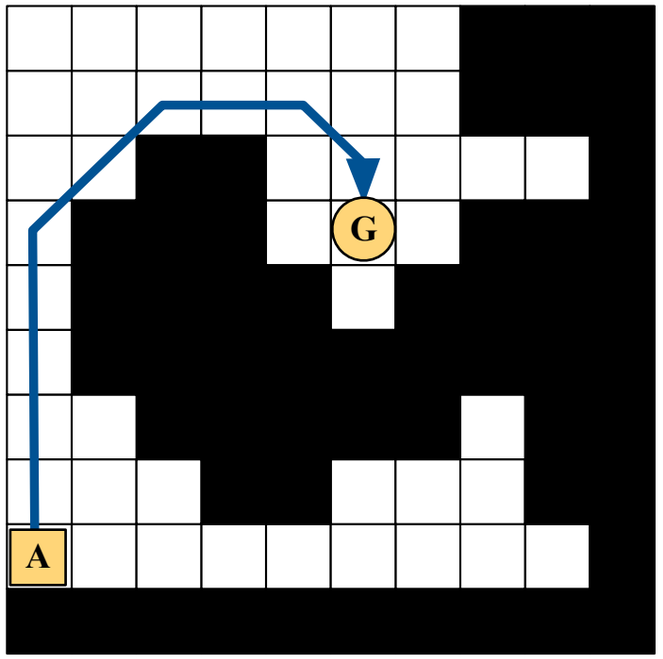
Theorem: Any pathfinding problem can be reduced into a canonical problem (agent size = 1, capability = 1 terrain)




Initial problem




Reduced size




Reduced size and capability

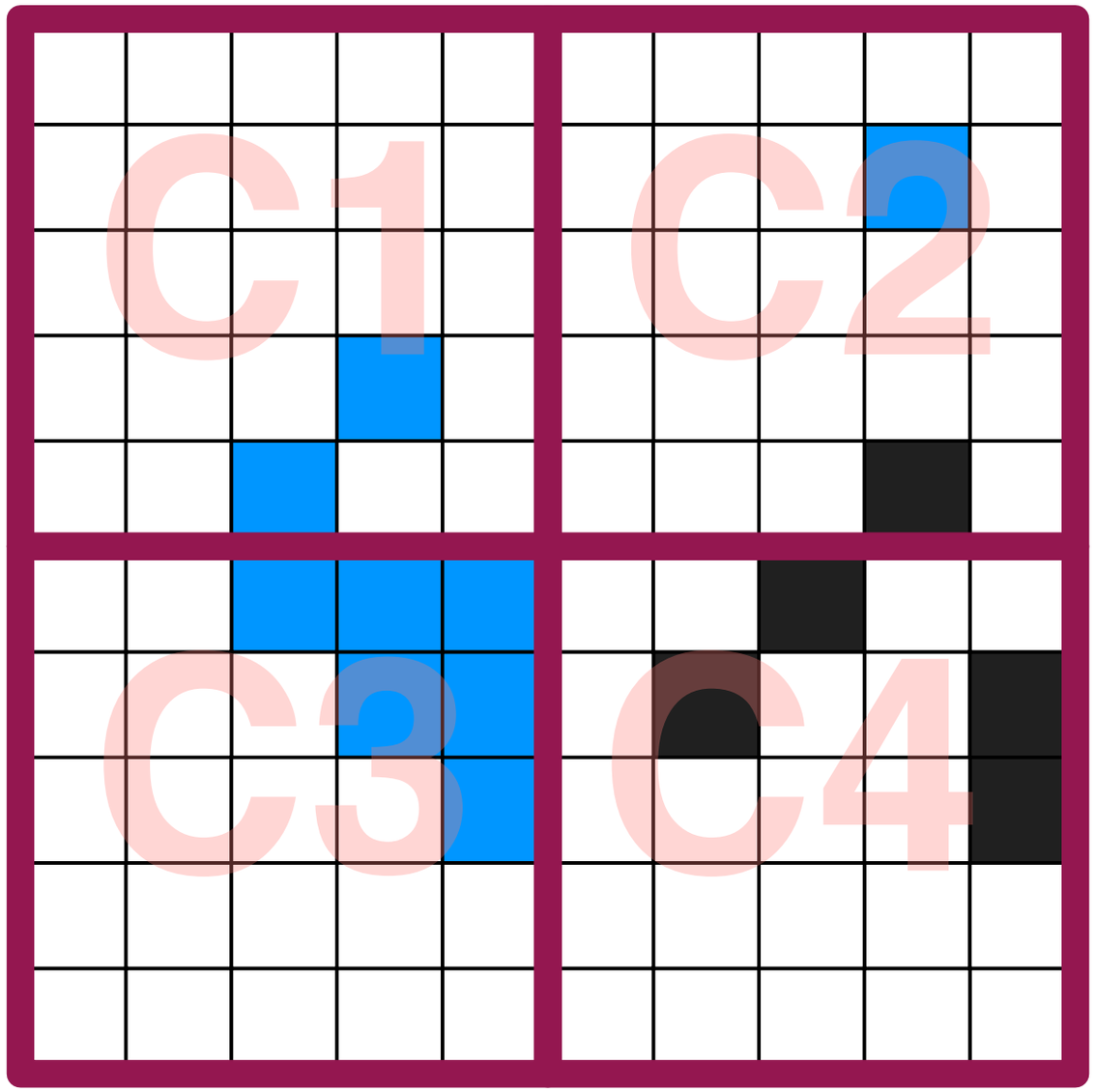
# Annotated A*

- Search process:
  - Similar to A*.
  - Extra parameters: Agent's size and capability.
  - Only expand nodes with clearance > agent size.
- Works great!
- For small problem sizes...

# Abstraction Process

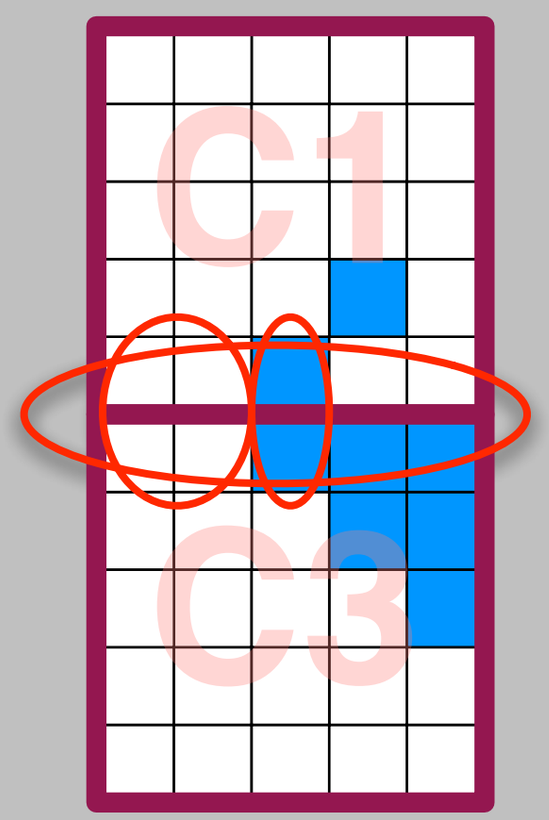
- Similar to HPA*:
  - Divide map into NxN clusters.
  - Identify entrances for all capabilities.
  - Using AA*, find intra-edges for all agent sizes and capabilities
  - Reduce the graph using dominance

# Clusters

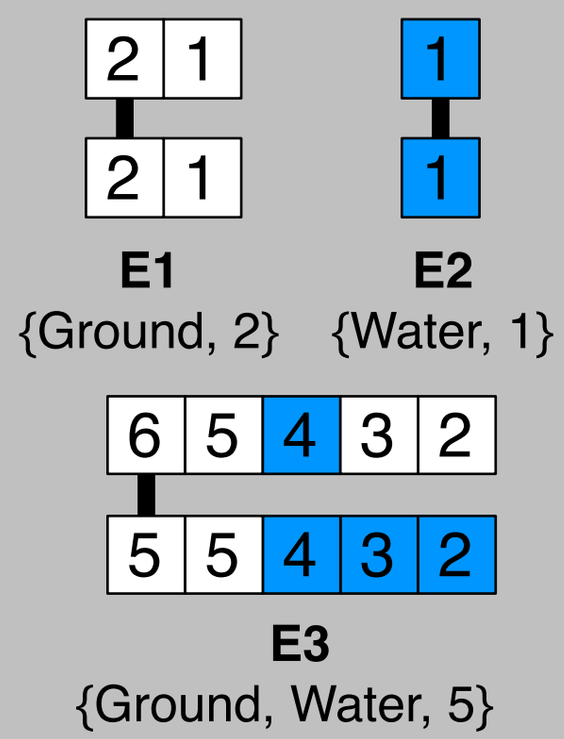


# Entrances

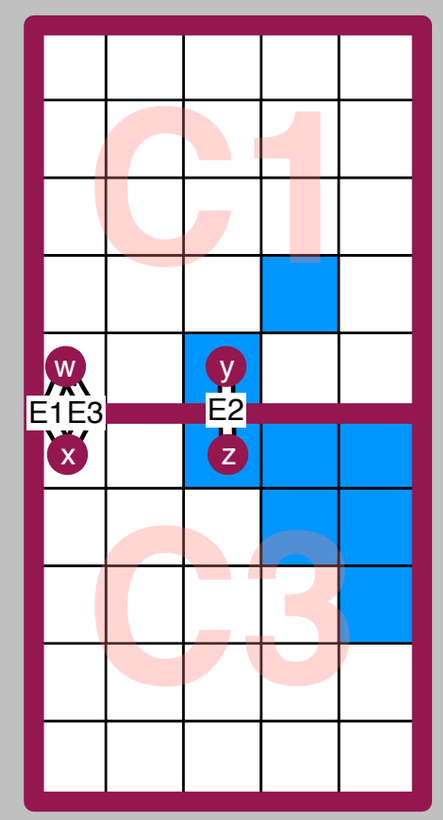
## Identifying entrances




## Transition points

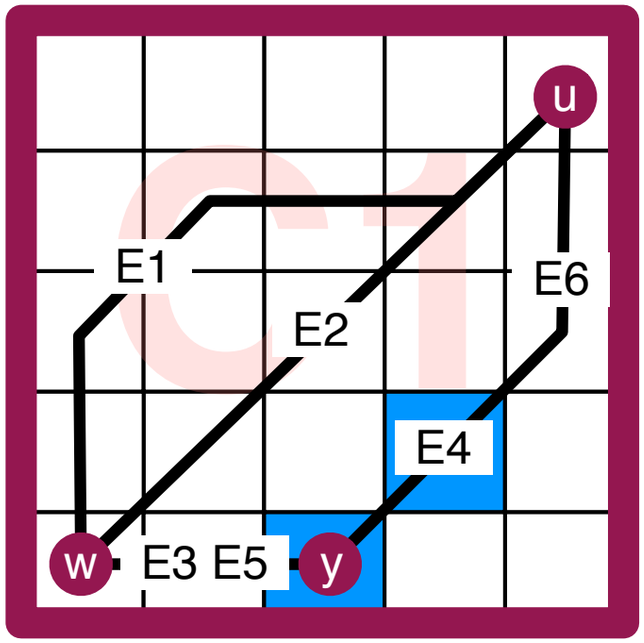



## Abstract graph



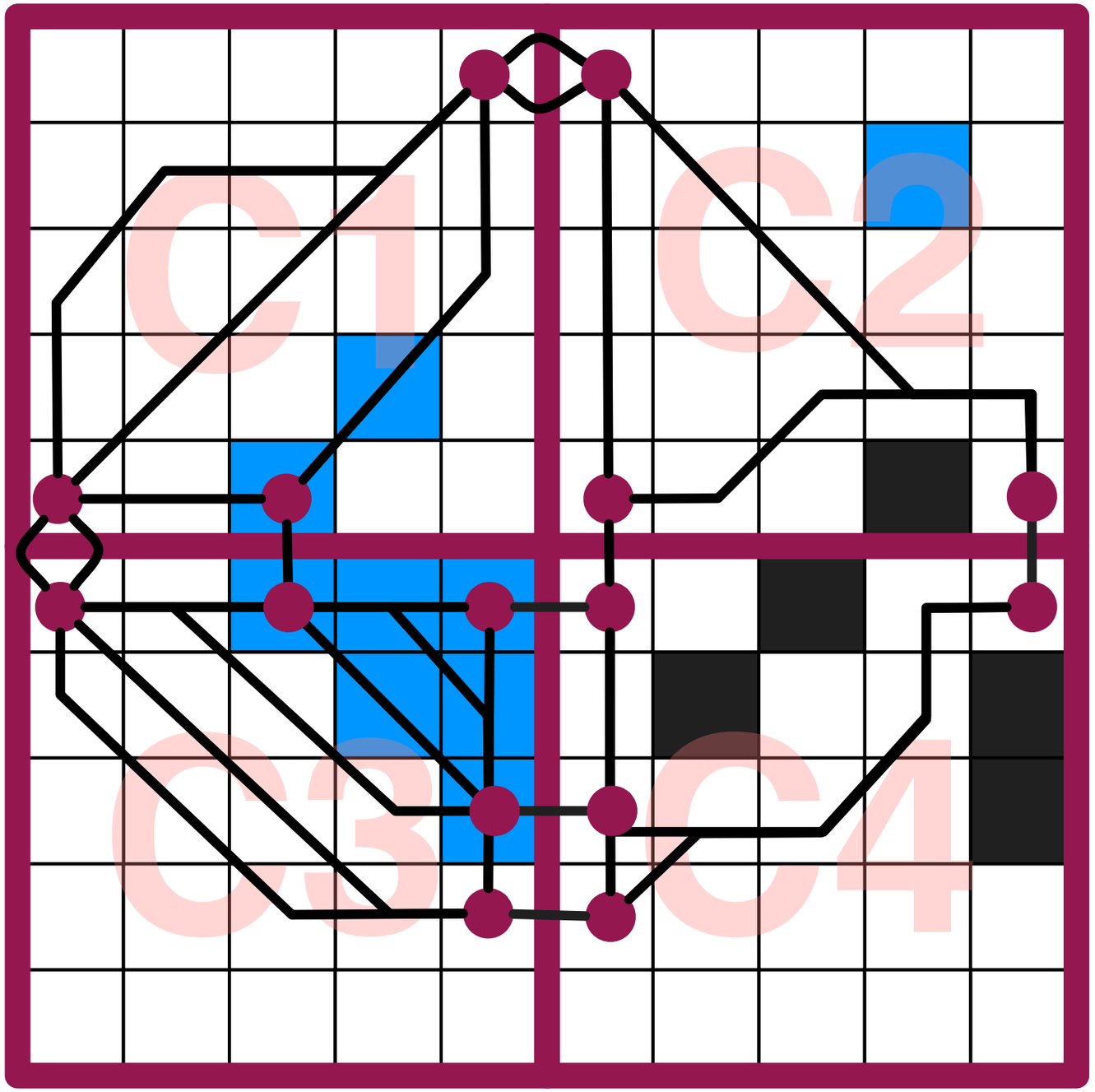
# Intra-edges

- Use AA* to find all possible ways (sizes/capabilities) to traverse across each cluster.
- For each path found, add a new abstract edge.




Edge Annotations
{Terrain, Clearance}:
E1 = {Ground, 2}
E2 = {Ground, 1}
E3 = {Ground, Water, 2}
E4 = {Ground, Water, 2}
E3 = {Ground, Water, 1}
E4 = {Ground, Water, 1}

# Initial abstraction

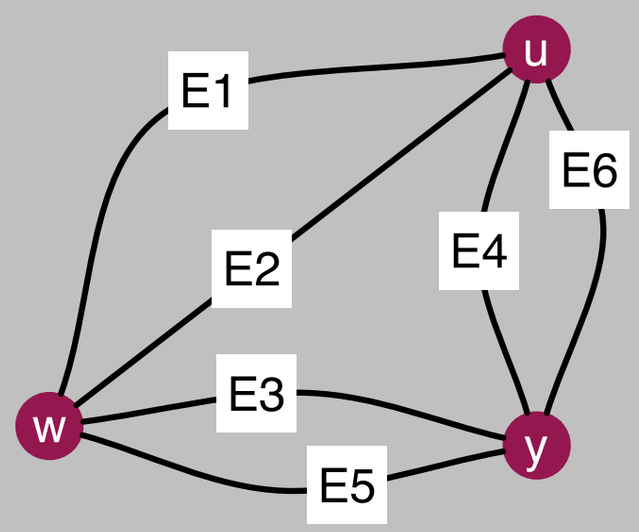


# Compacting the abstract graph

- Method produces a representationally complete graph but can get rather large.
- Solutions:
  - Strong dominance
  - Weak dominance

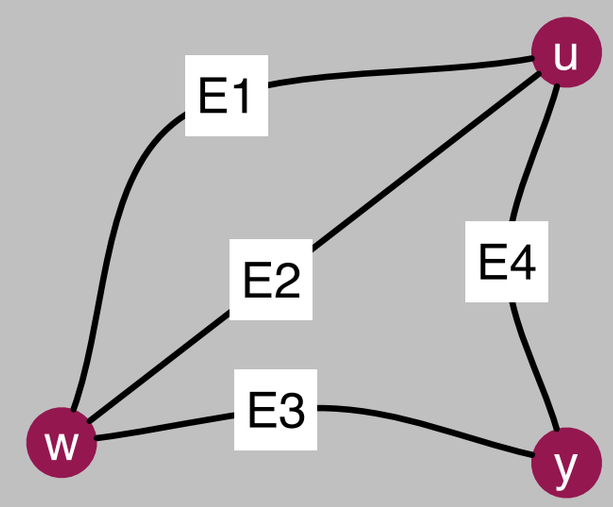
# Strong dominance

## Initial graph




Edge Annotations
[Capability, Clearance, Length]:
E1 = [{Ground}, 2, 7.5]
E2 = [{Ground}, 1, 4.5]
E3 = [{Ground, Water}, 2, 3.0]
E4 = [{Ground, Water}, 2, 3.0]
E5 = [{Ground, Water}, 1, 3.0]
E6 = [{Ground, Water}, 1, 3.0]

## High Quality graph

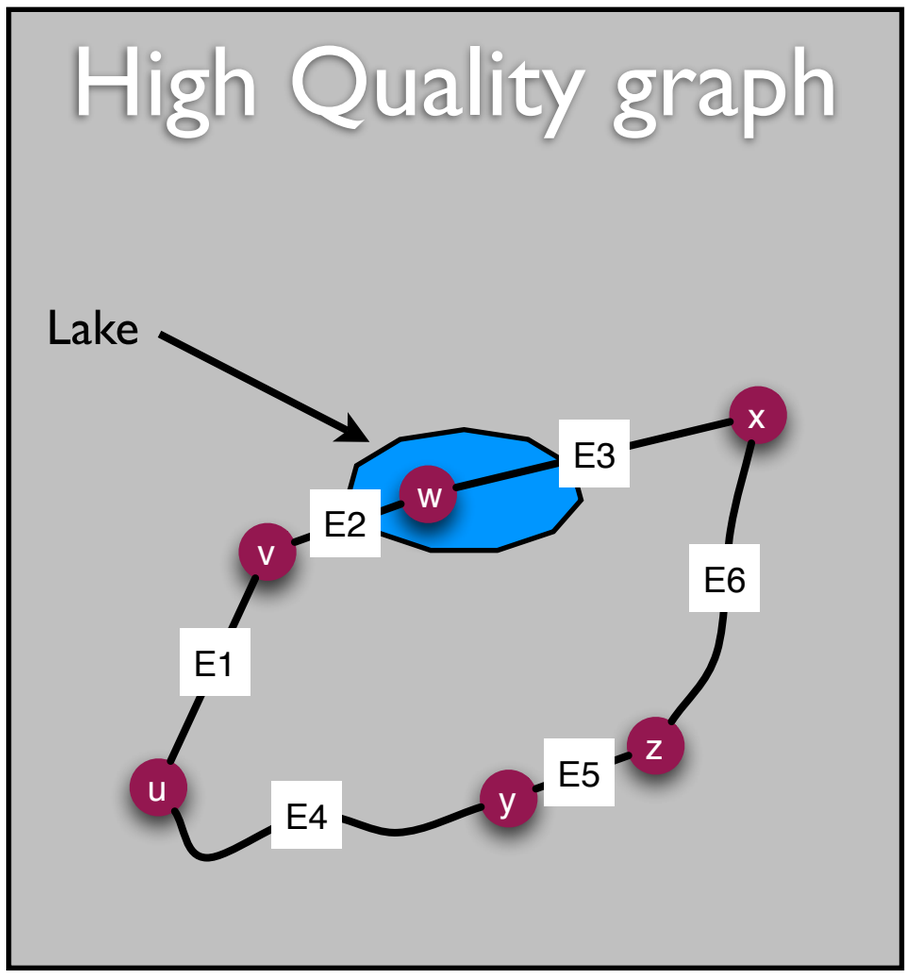


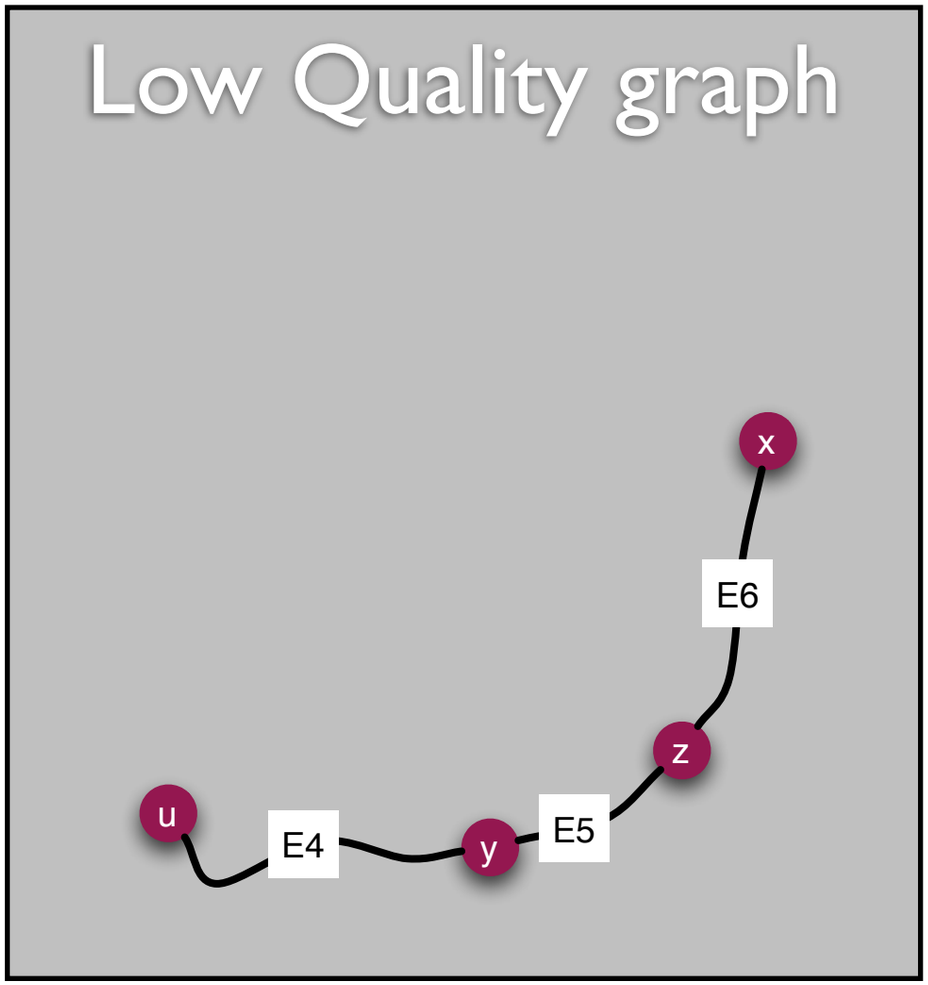

Edge Annotations
[Capability, Clearance, Length]:
E1 = [{Ground}, 2, 7.5]
E2 = [{Ground}, 1, 4.5]
E3 = [{Ground, Water}, 2, 3.0]
E4 = [{Ground, Water}, 2, 3.0]

Remove edges with smaller clearance (all else being identical)

# Weak dominance

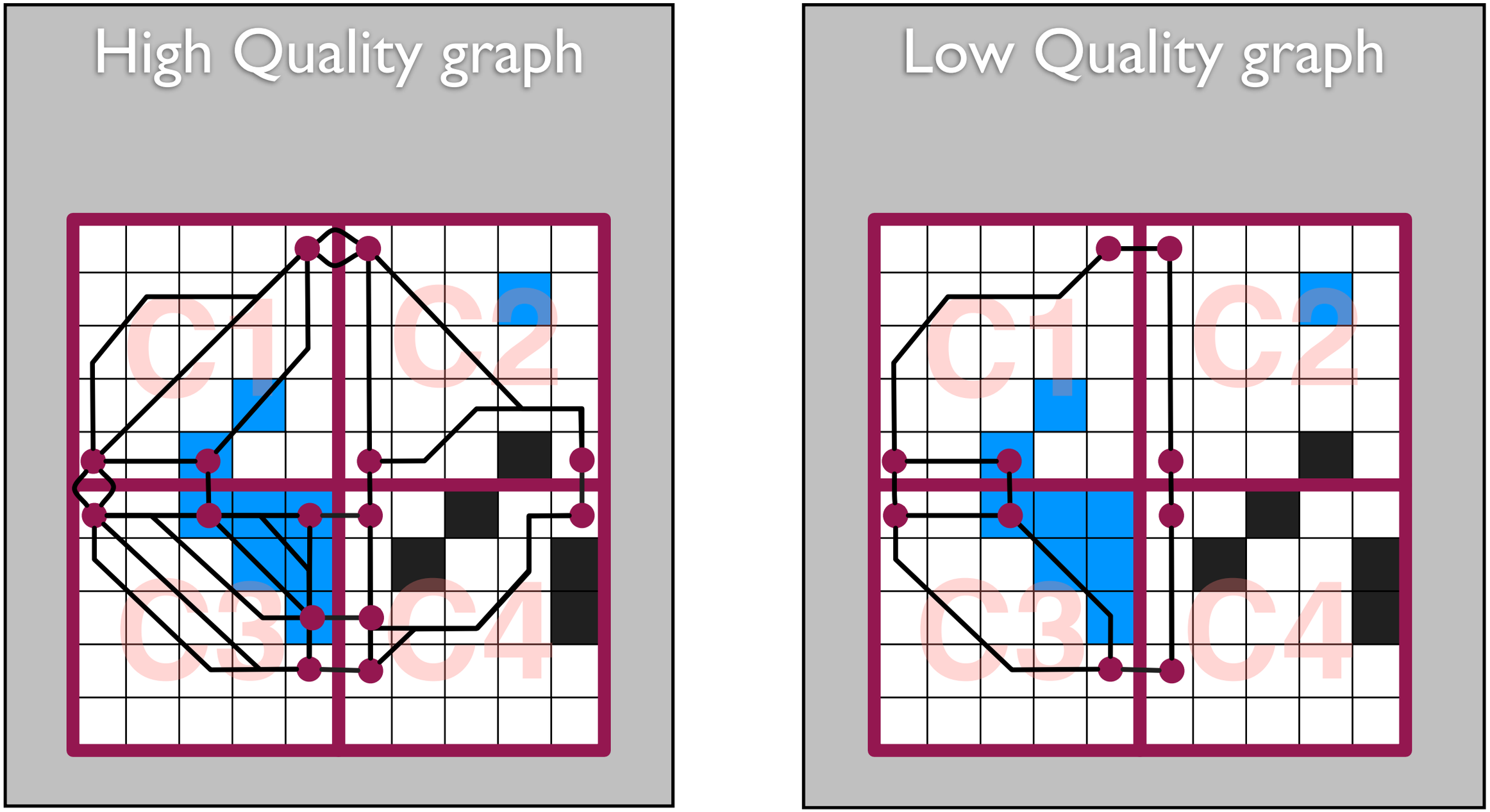
Retain only edges traversable by largest number of agents (aka. prefer freeways where possible)




Two places of interest: U and X.
Path p = {E1, E2, E3}. Length: 15, Max Size = 1, Capability: {Ground, Water}
Path q = {E4, E5, E6}. Length: 17, Max Size = 2, Capability: {Ground}

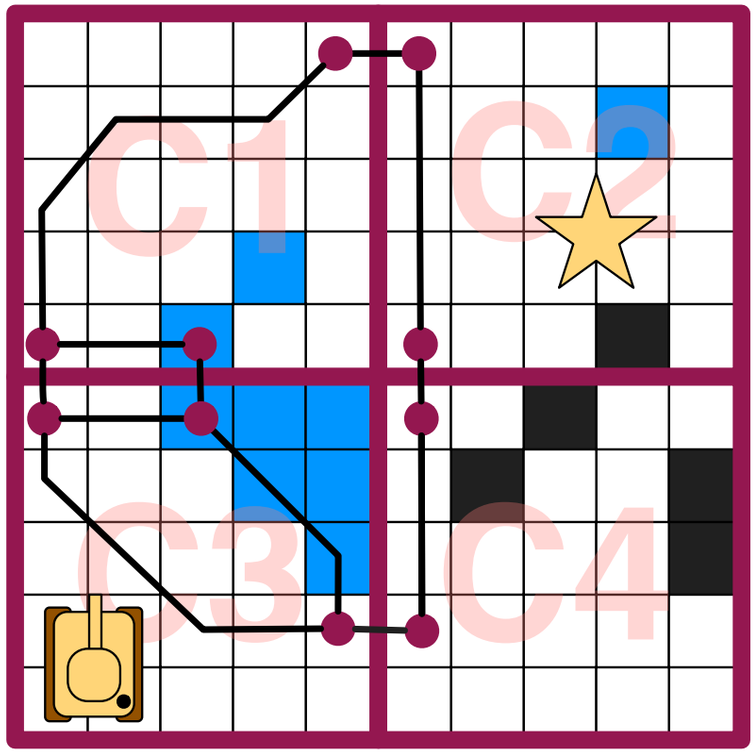
# Weak dominance



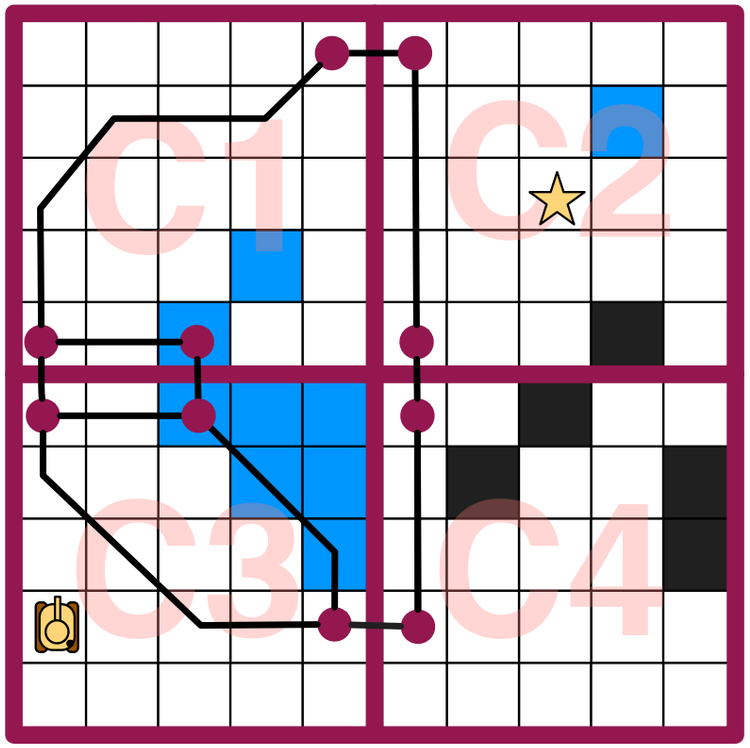

Weak dominance is applied to transitions between adjacent clusters (inter-edges).

# HAA*

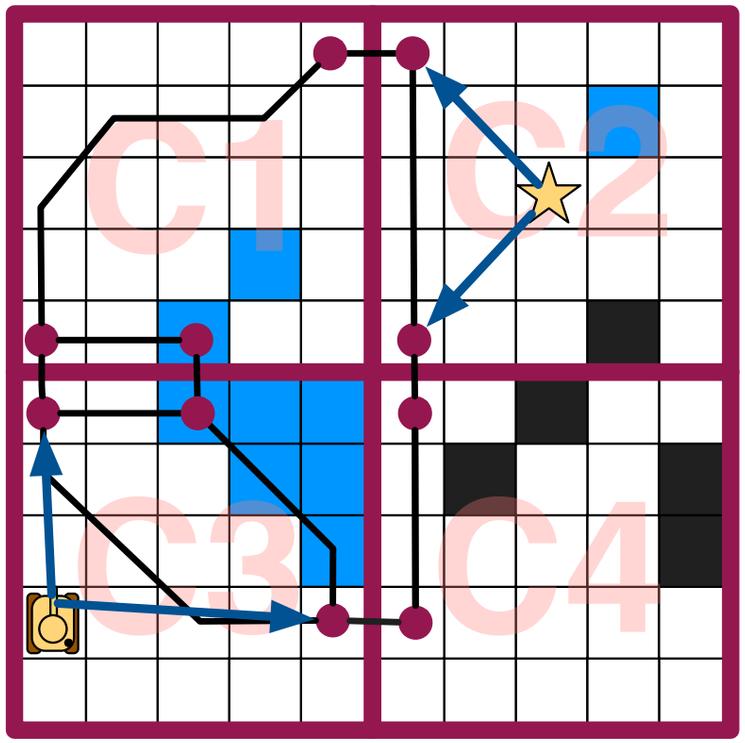
- Similar search process to HPA*
  - Insert Start + Goal
  - Find + refine abstract path
- Differences:
  - Add a node to open list only if reachable according to edge annotations.



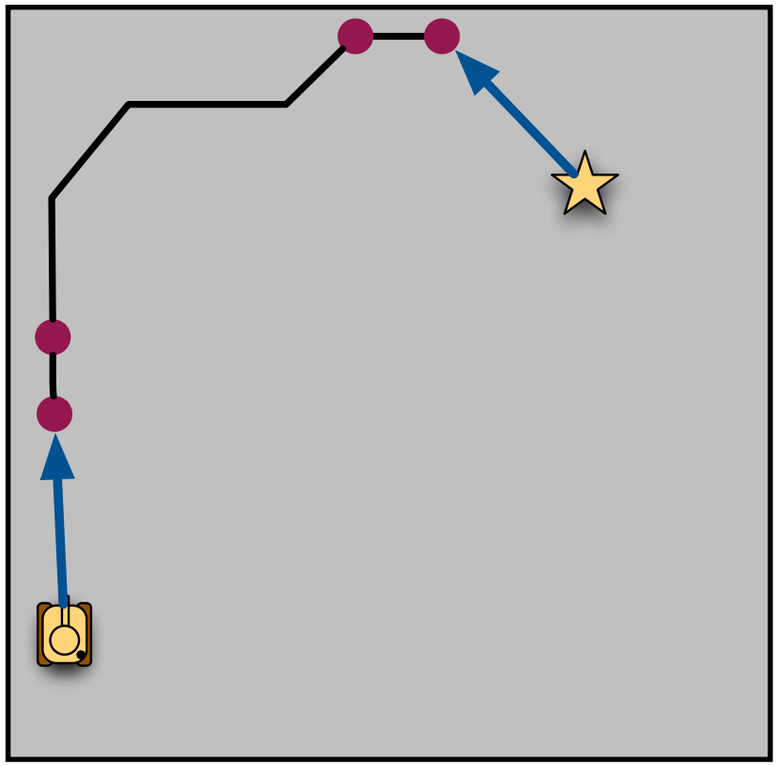

Initial problem
Size = 2, Capability = {Ground}




Reducing the problem




Inserting Start + Goal



Solution

# HAA* Advantages

- Works for units with different sizes and capabilities.
- Complete and near-optimal (within 4-8% from optimal on average)
- Fast (similar performance to HPA*)
- Low overhead (storing abstract graph requires little memory in practice)

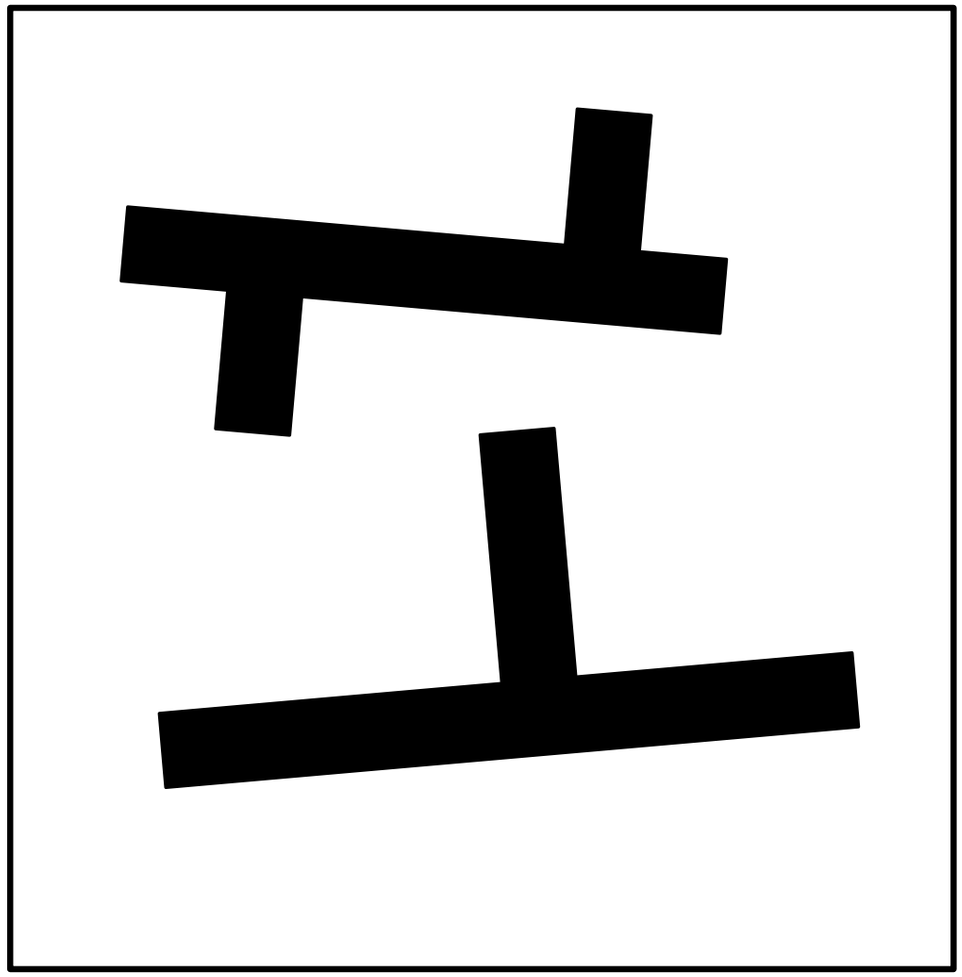
# HAA* Disadvantages

- Only works on grids.
- Need to model units using square bounding volumes.
- Insertion effort.

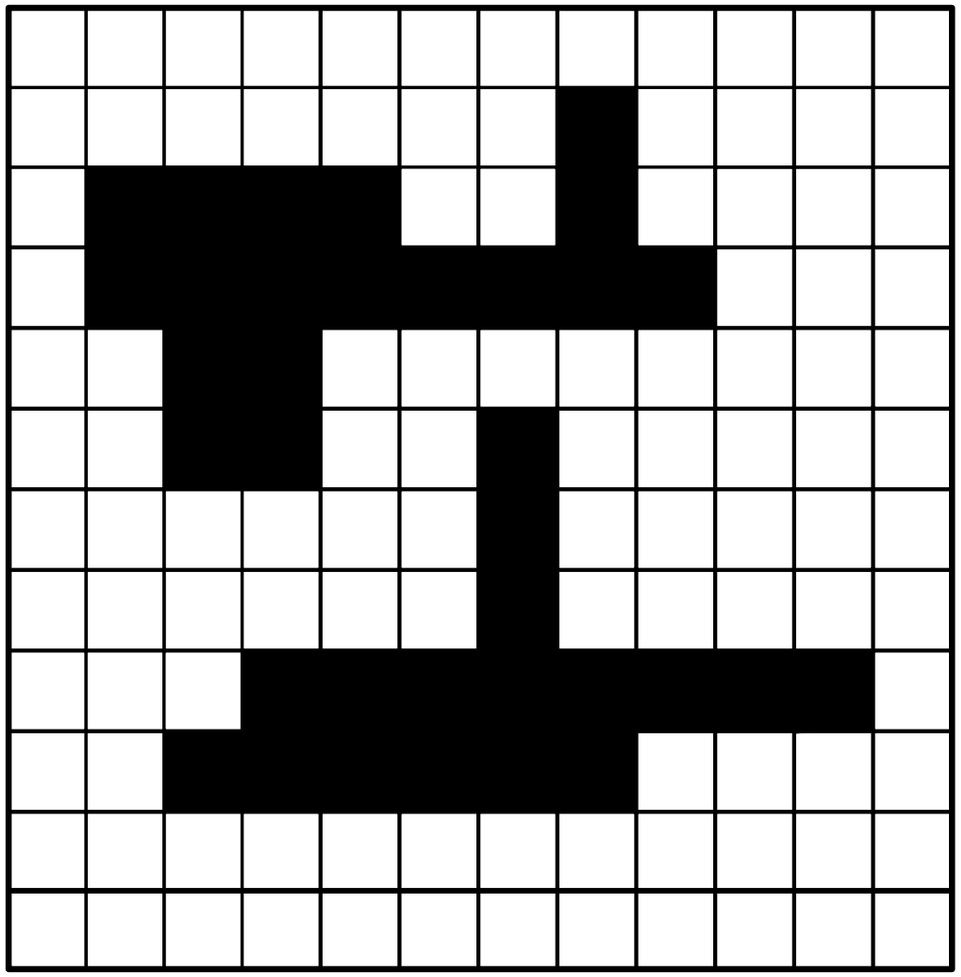
# Part 3:
# Probabilistic Roadmaps

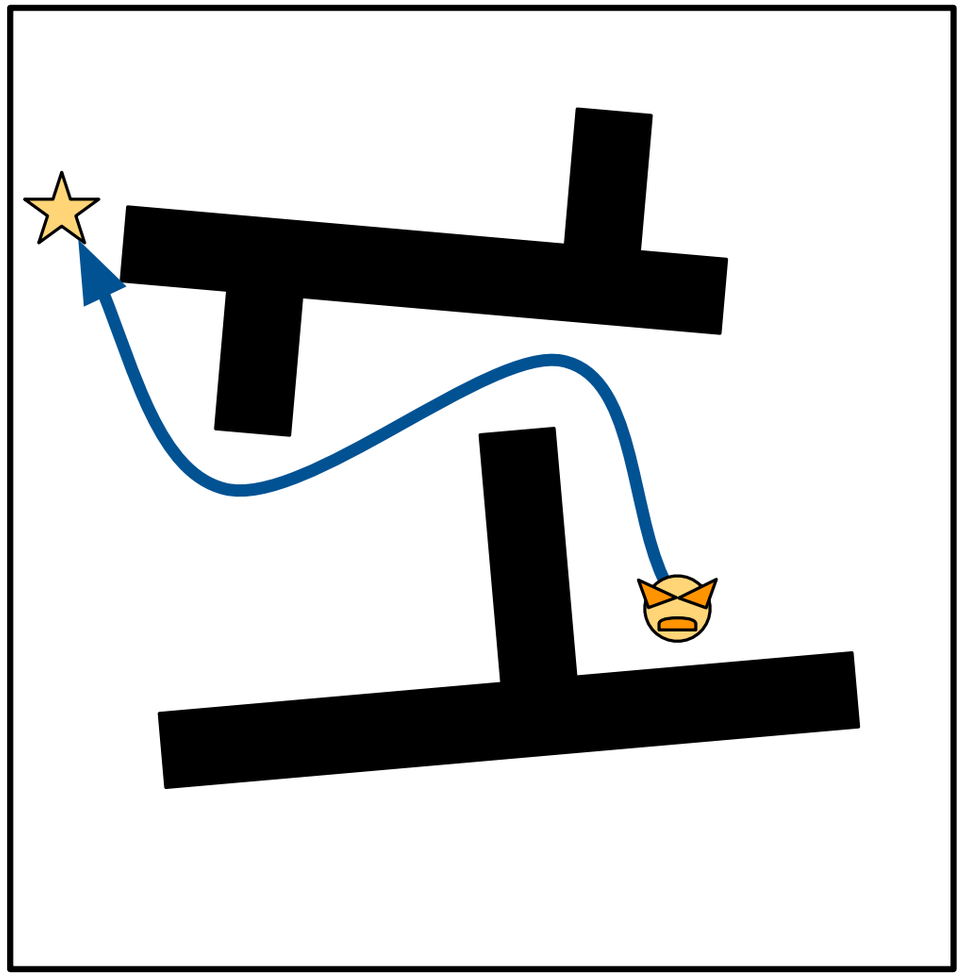
# Limitations of grid methods

- Coarse coverage of underlying terrain.
- Restricted movement (fixed angles)
- Paths don’t look realistic
- Often need to apply smoothing
- Not very useful in 3D spaces

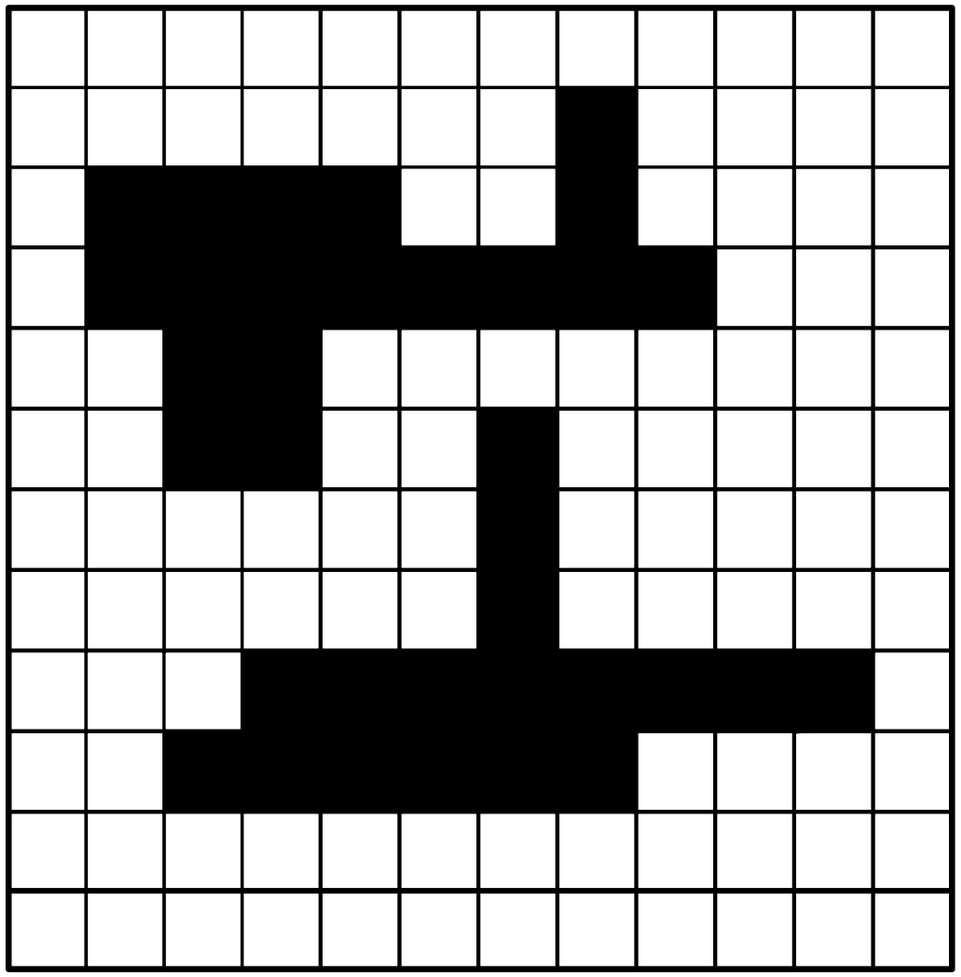


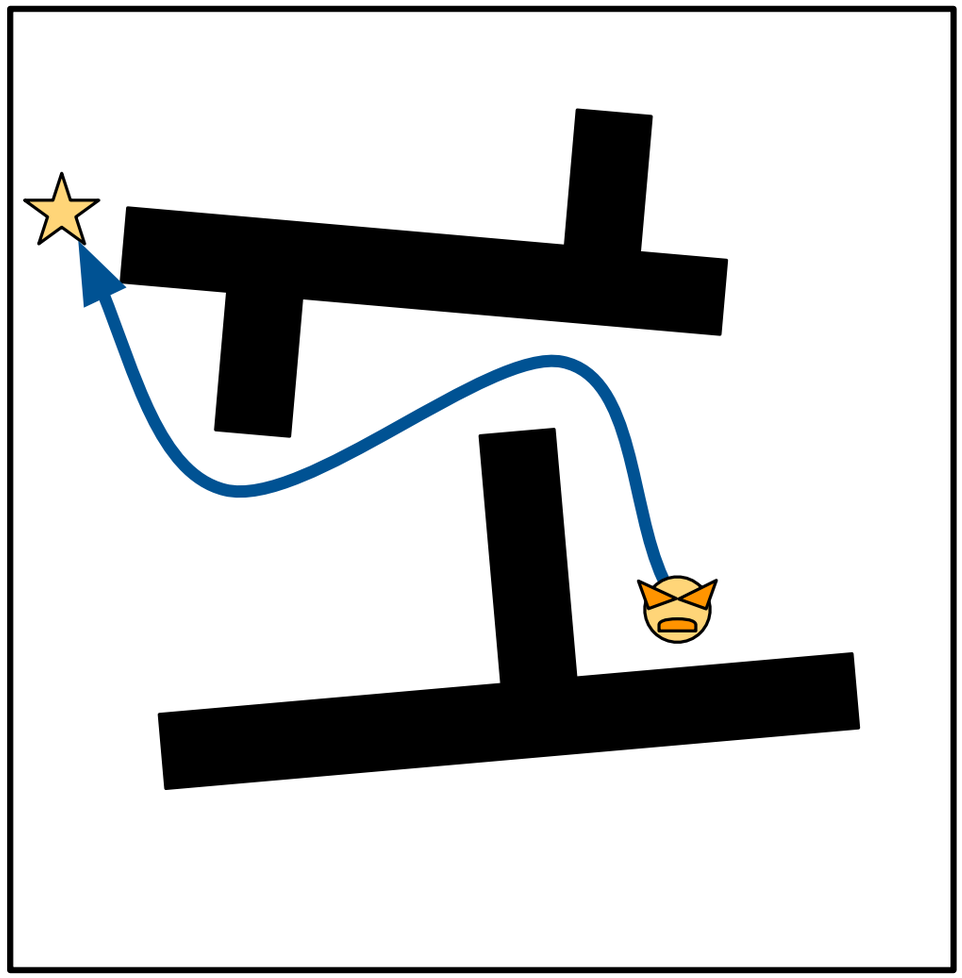
vs.



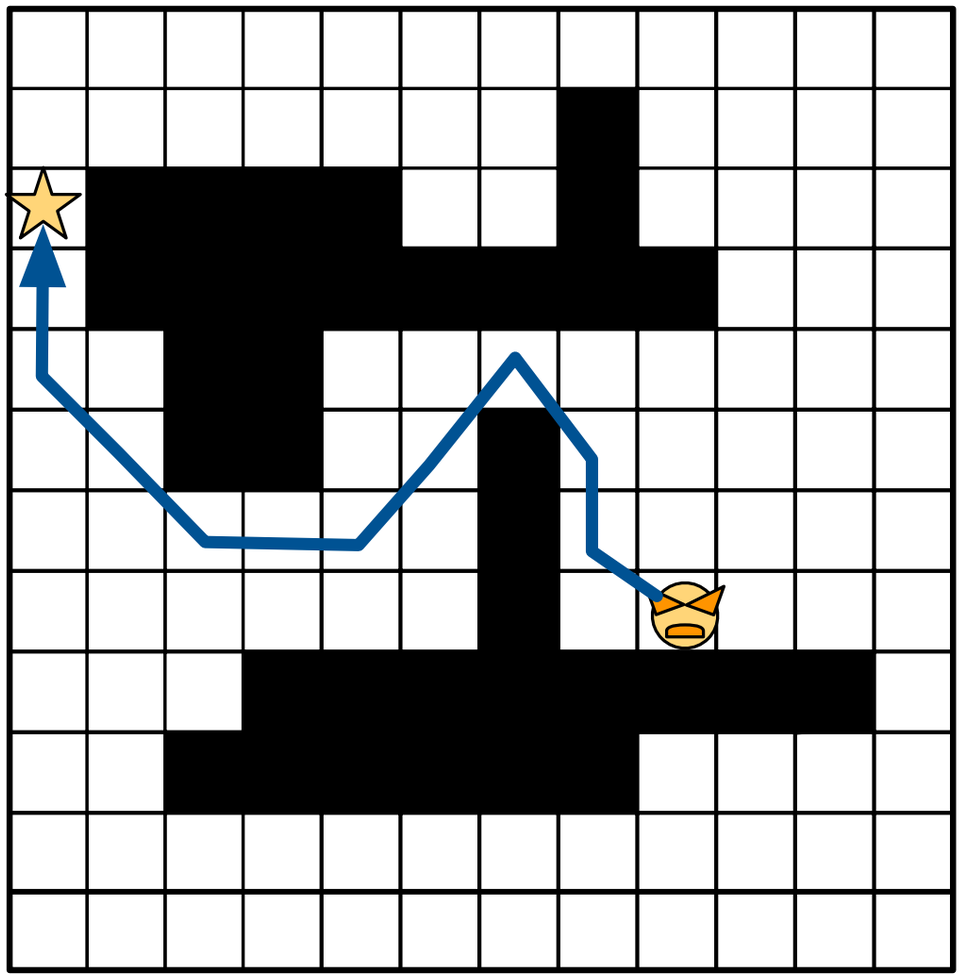


vs.



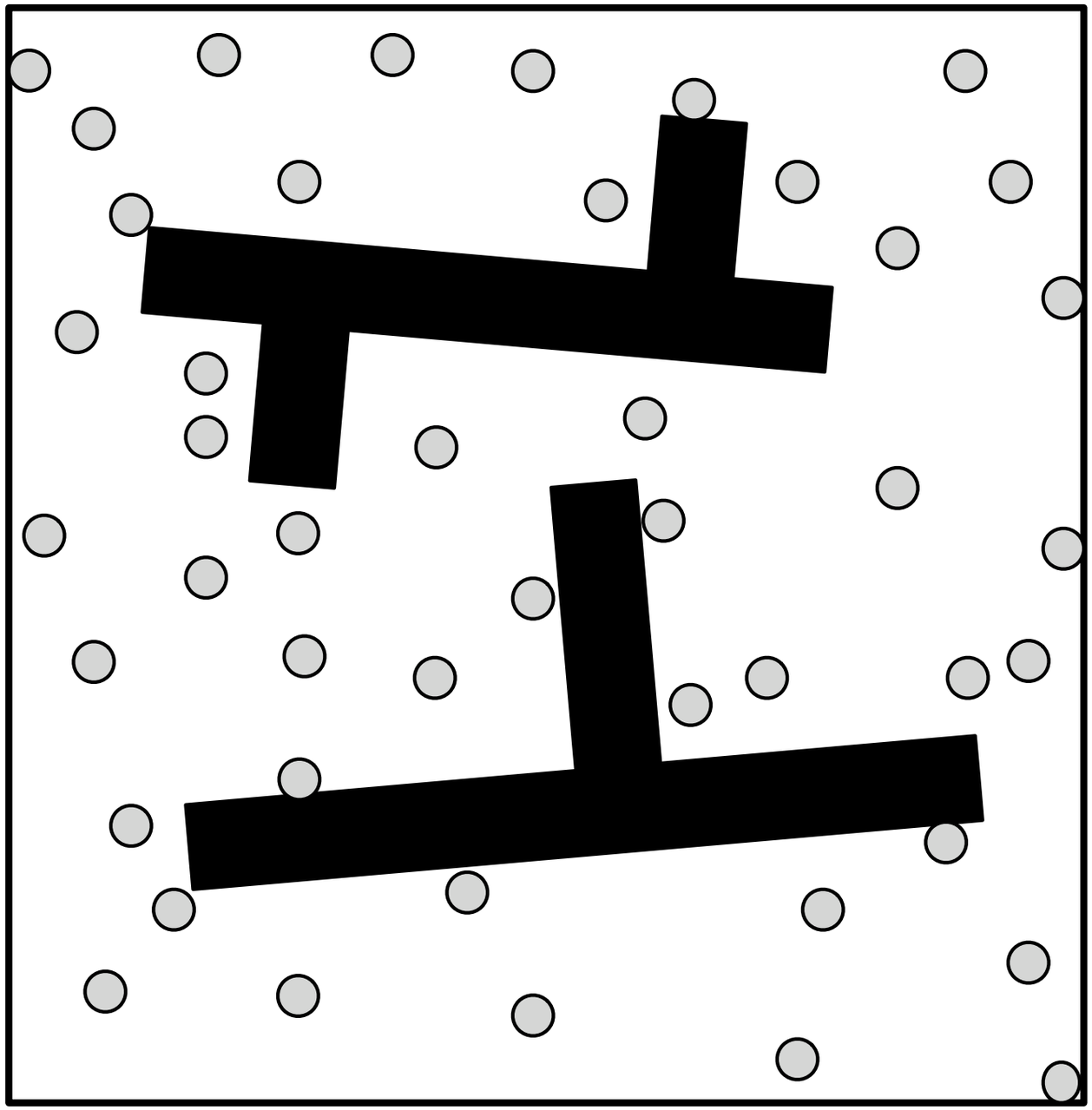


vs.

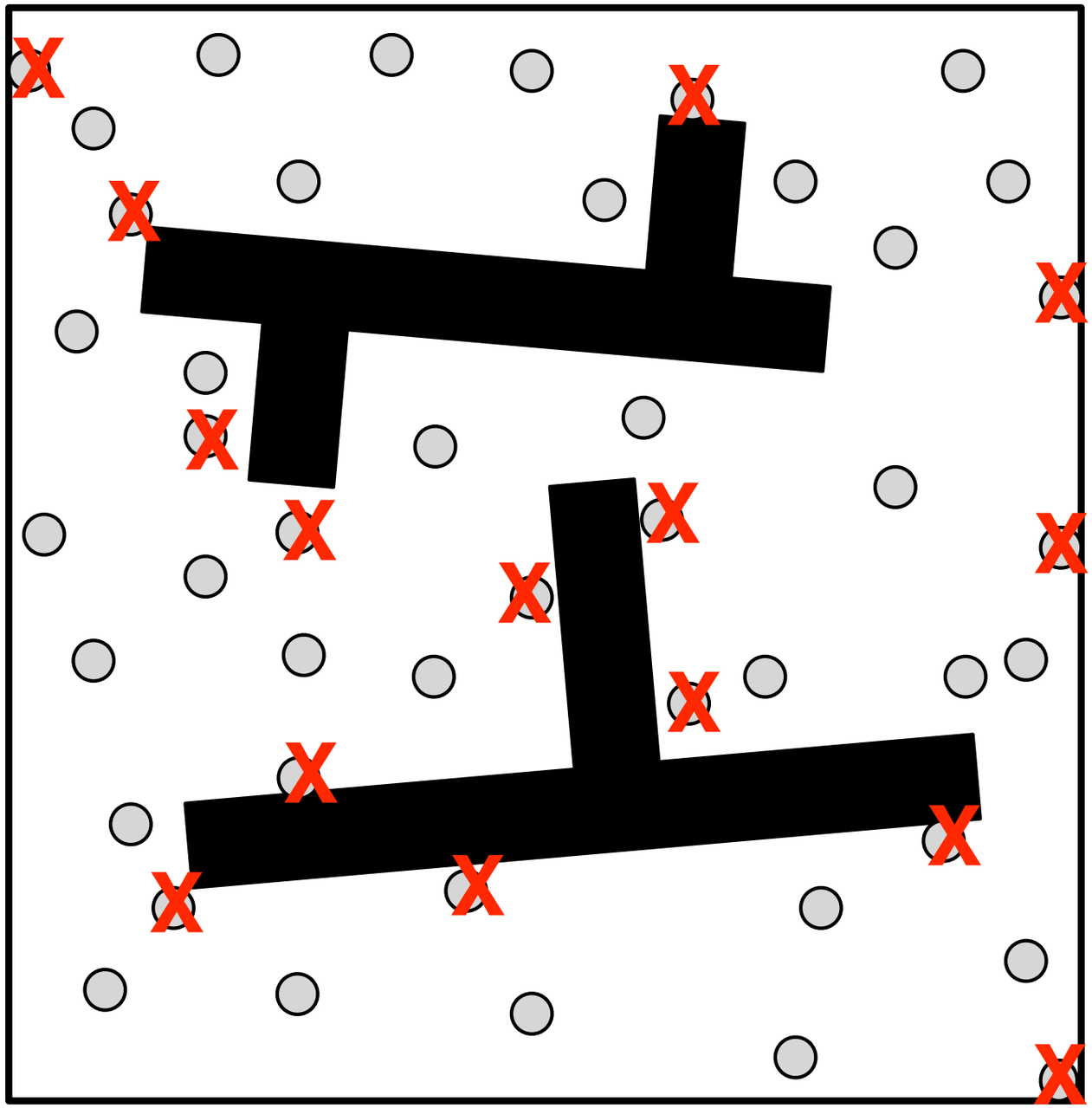


# Probabilistic Road Maps

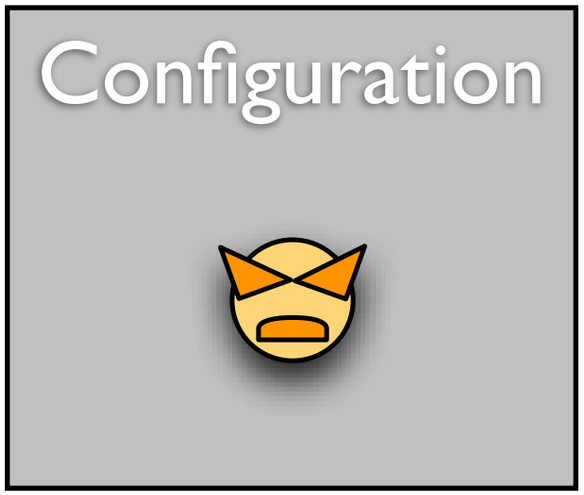
- PRM: Robotics inspired method from mid 90s.
  e.g. see [Choset et al, 2004].
- Pick a robot configuration.
- Build a graph by taking lots of sample points.

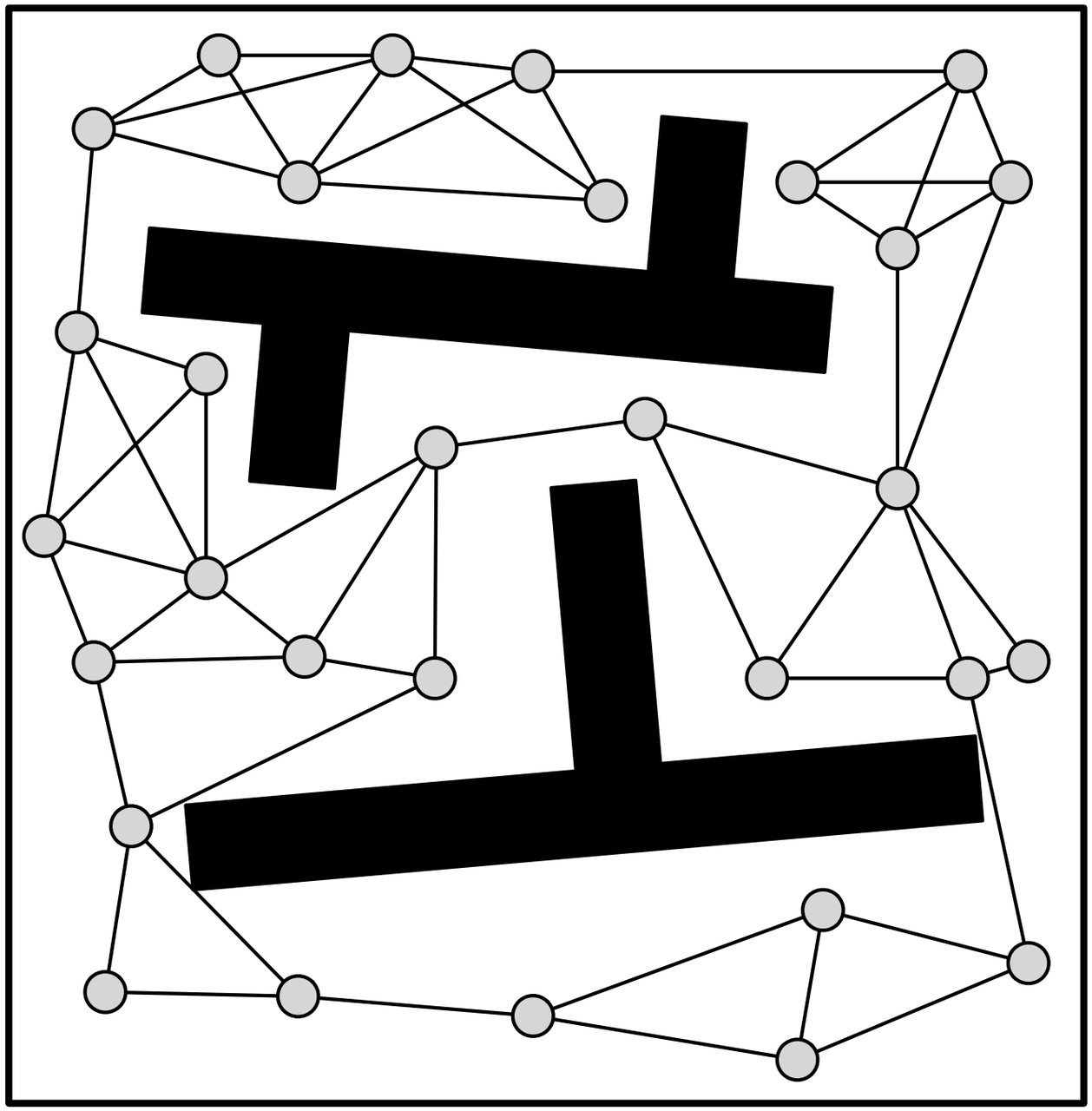


Select non-obstacle sample points from the map

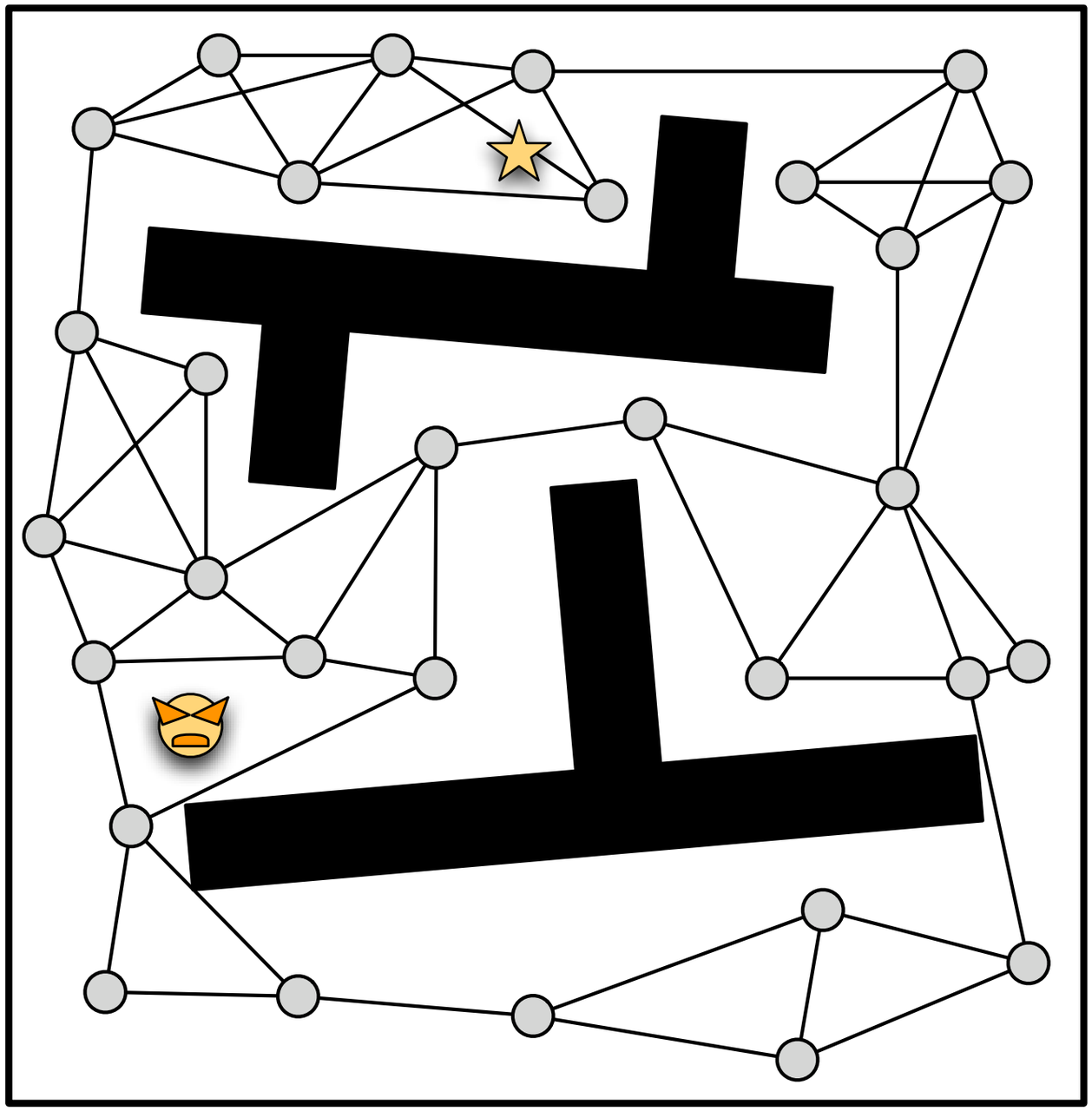


Retain collision-free points for some configuration of an agent

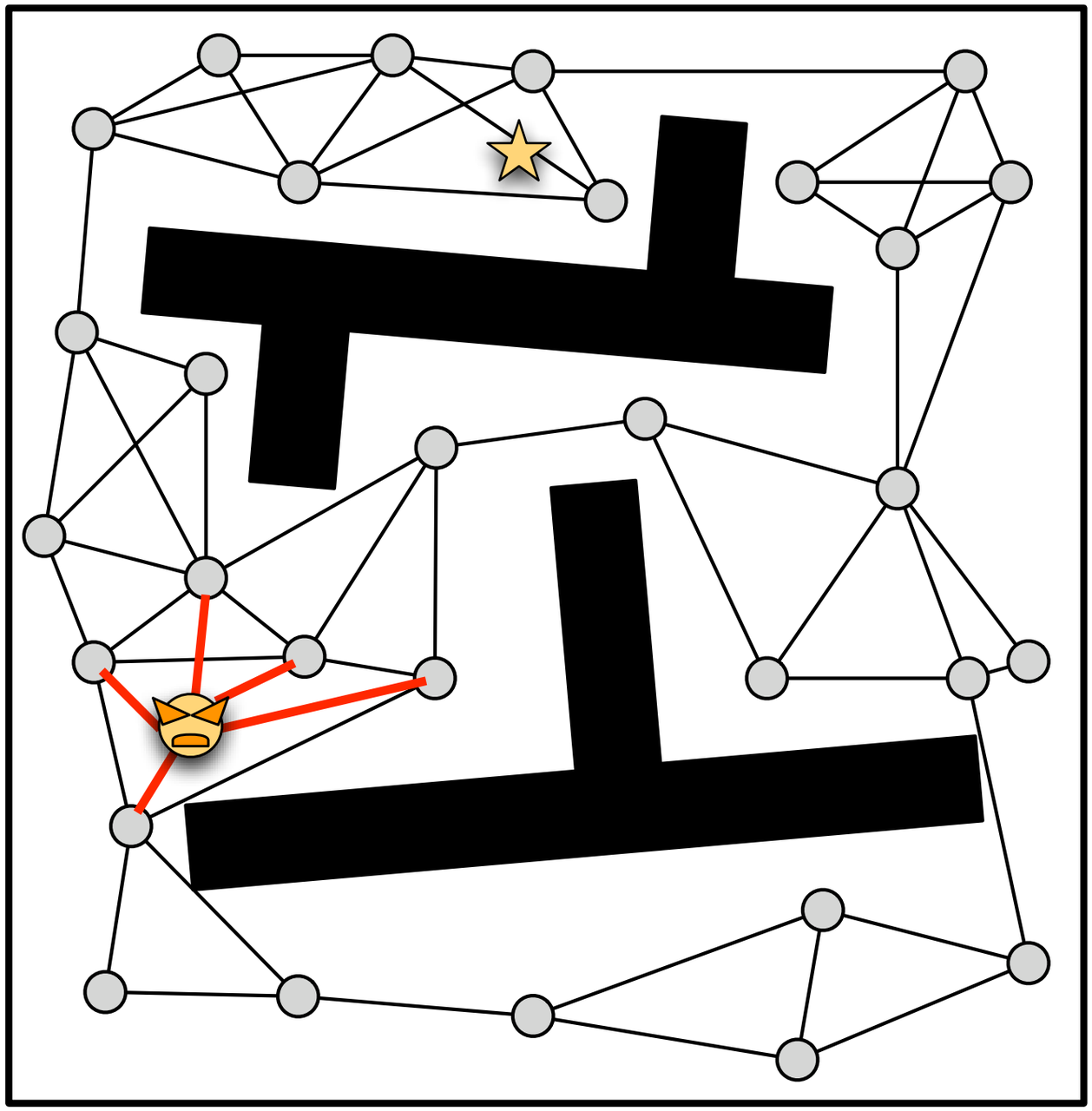




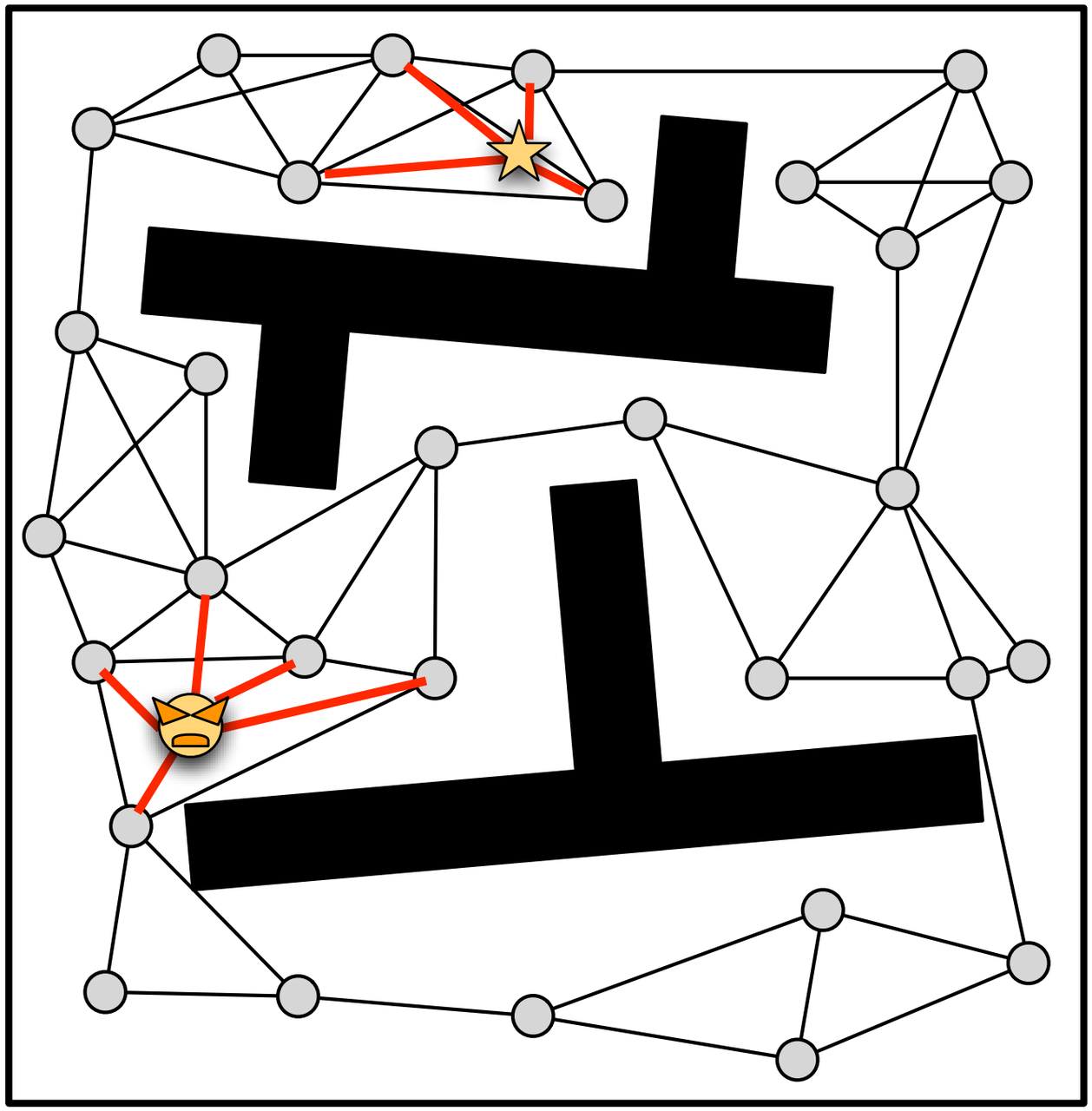
Connect nearby points to form a graph



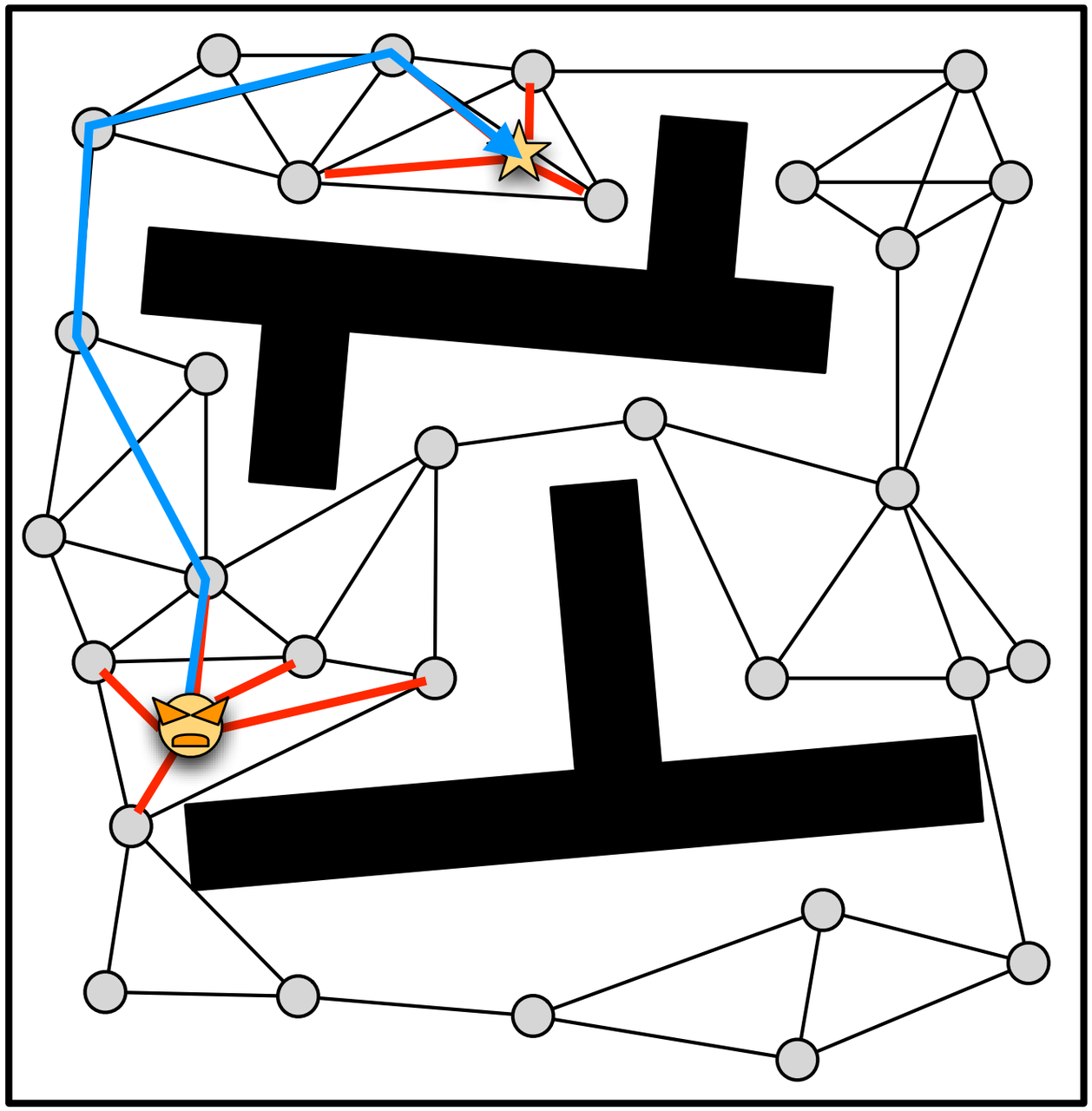
Pathfinding: Connect start and goal to rest of roadmap



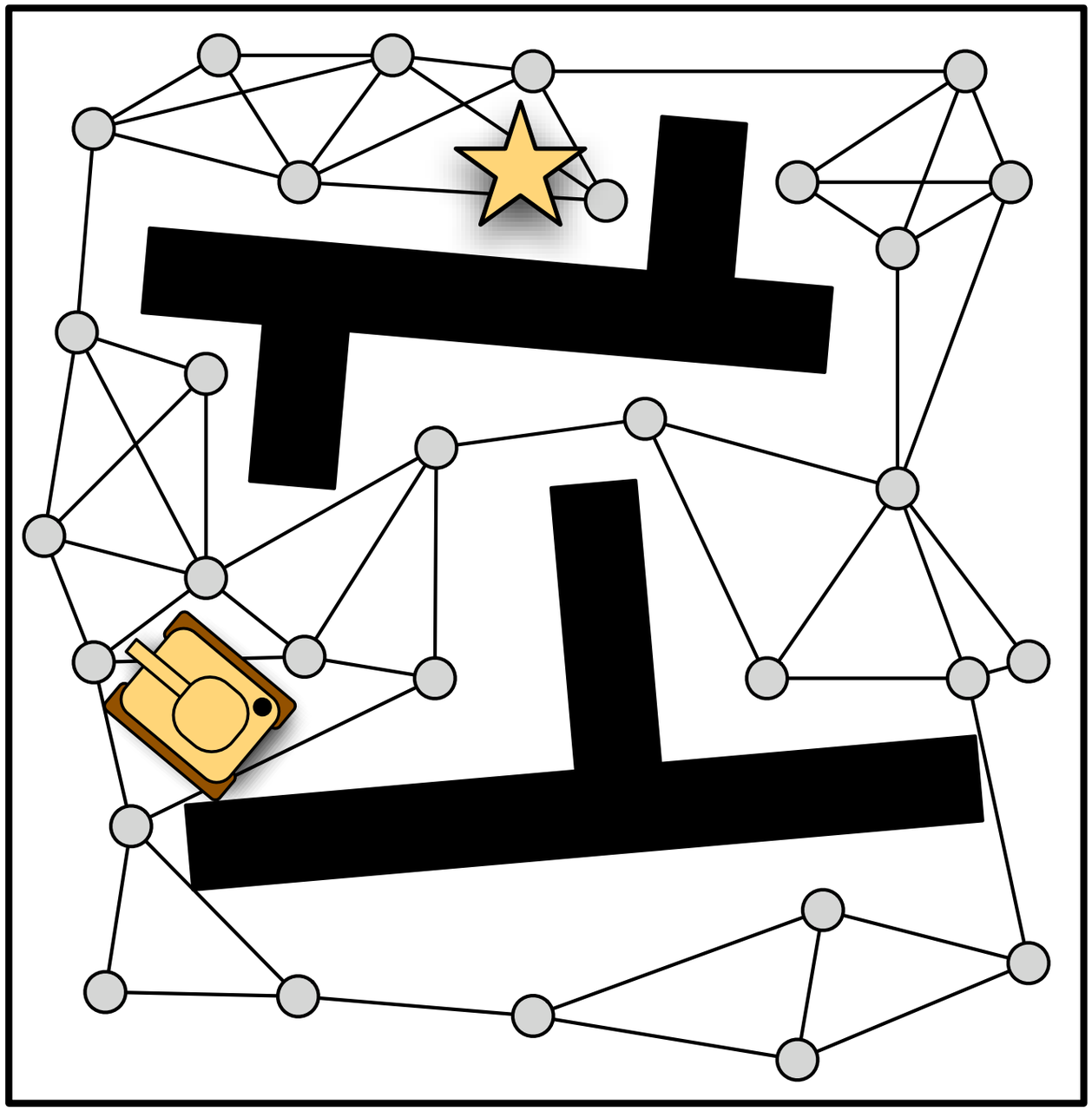
Pathfinding: Connect start and goal to rest of roadmap



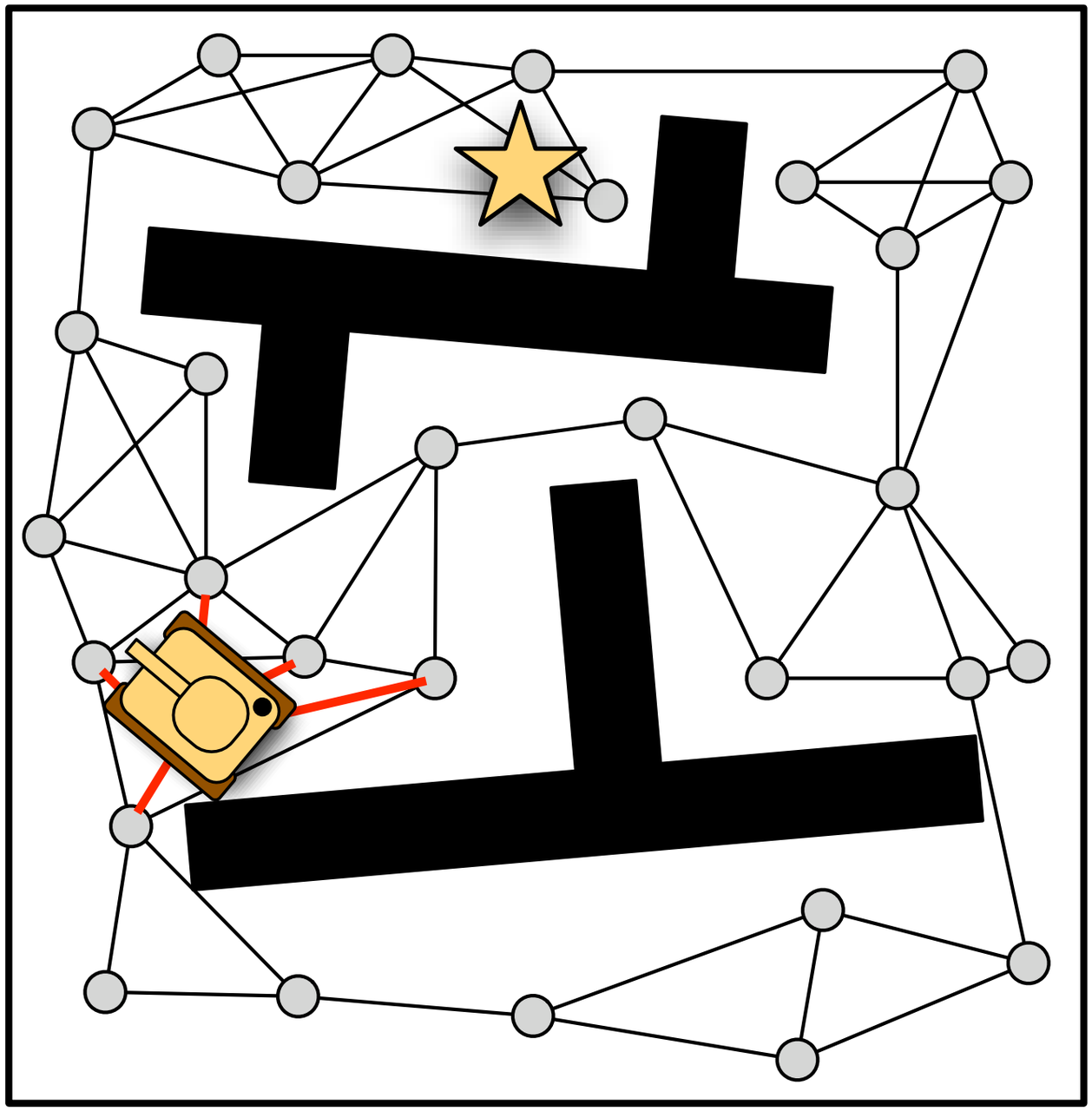
Pathfinding: Connect start and goal to rest of roadmap



Pathfinding: Connect start and goal to rest of roadmap



Most PRMs are only useful for agent/configuration used during construction.



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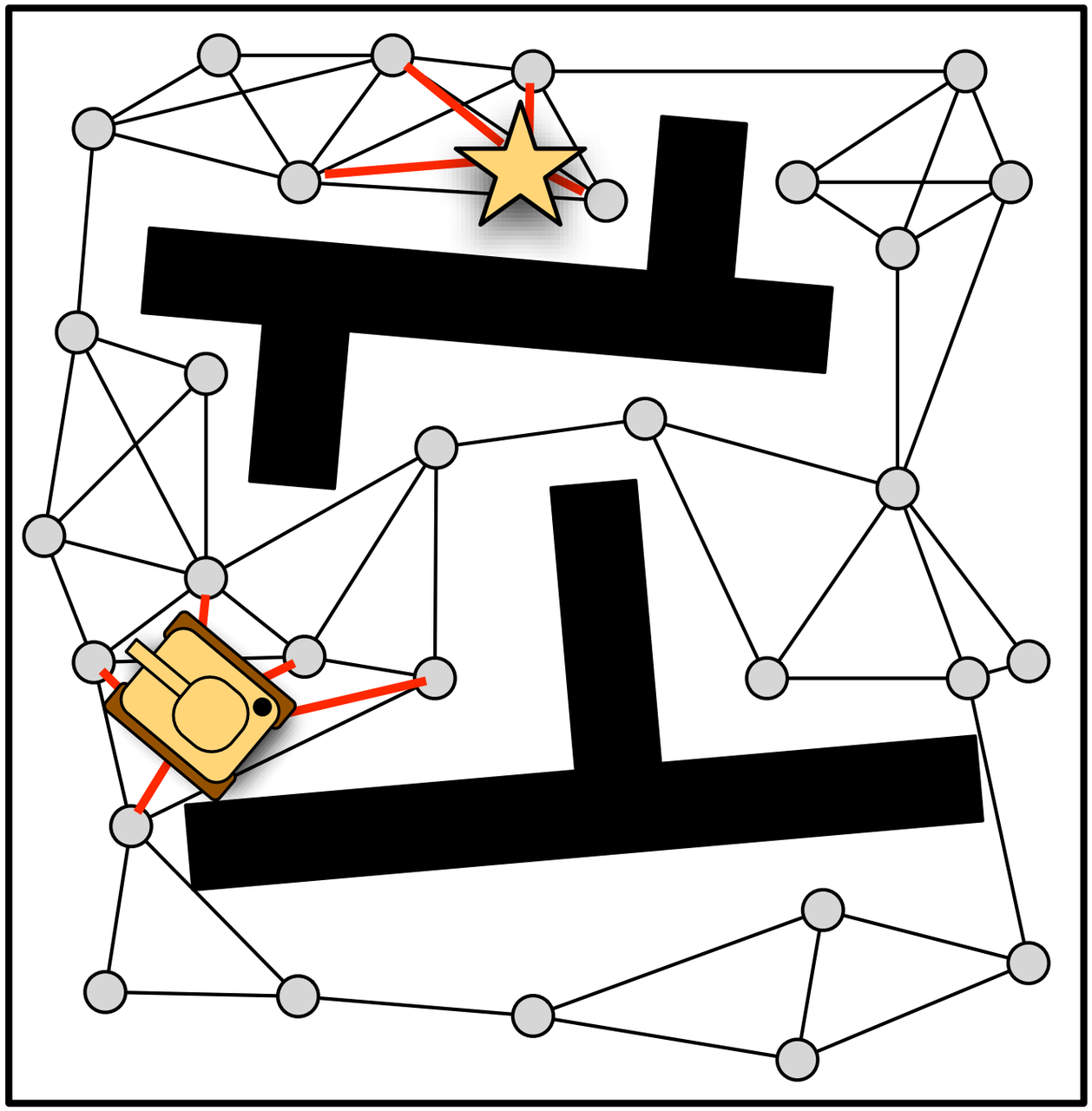

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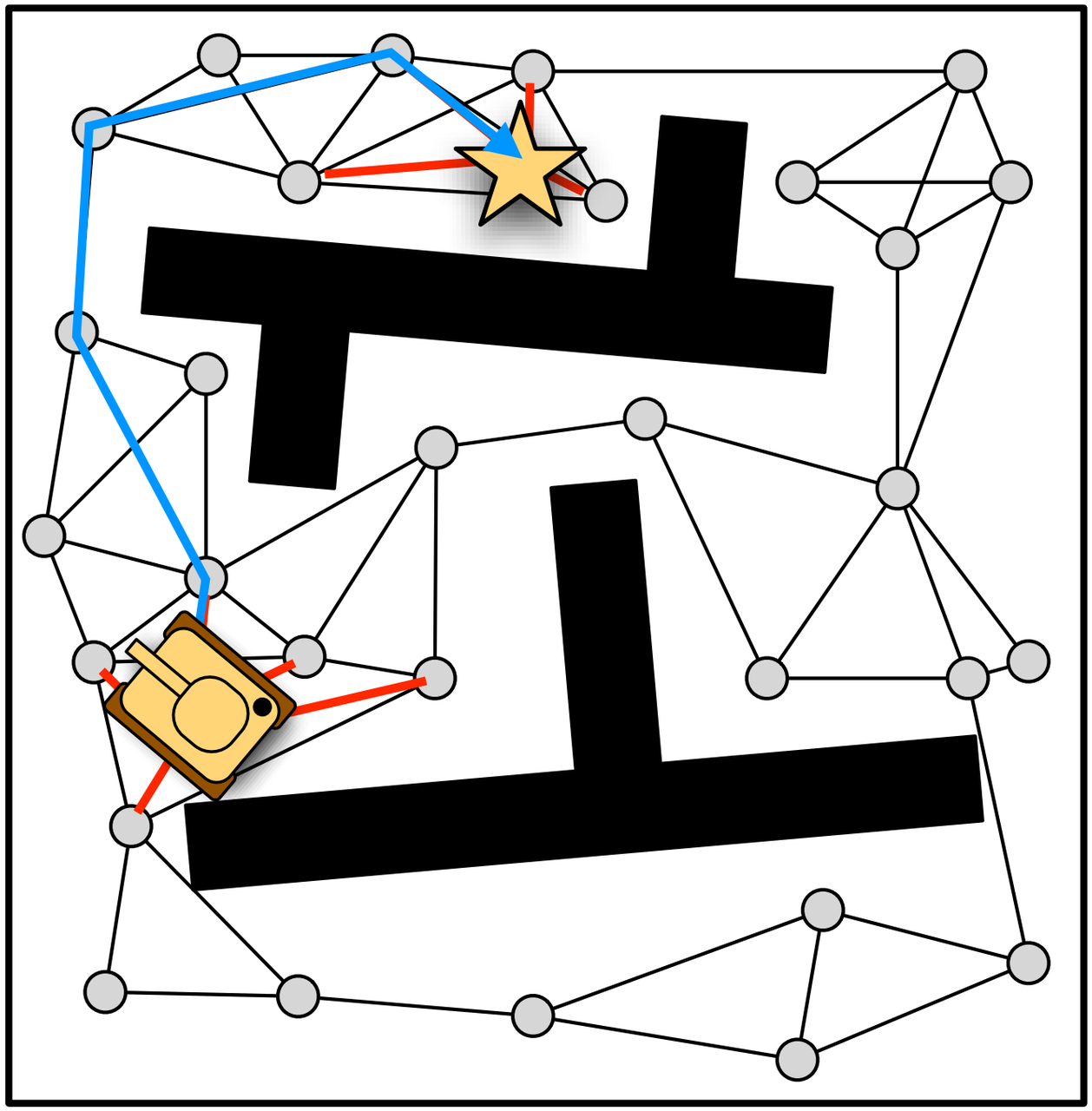

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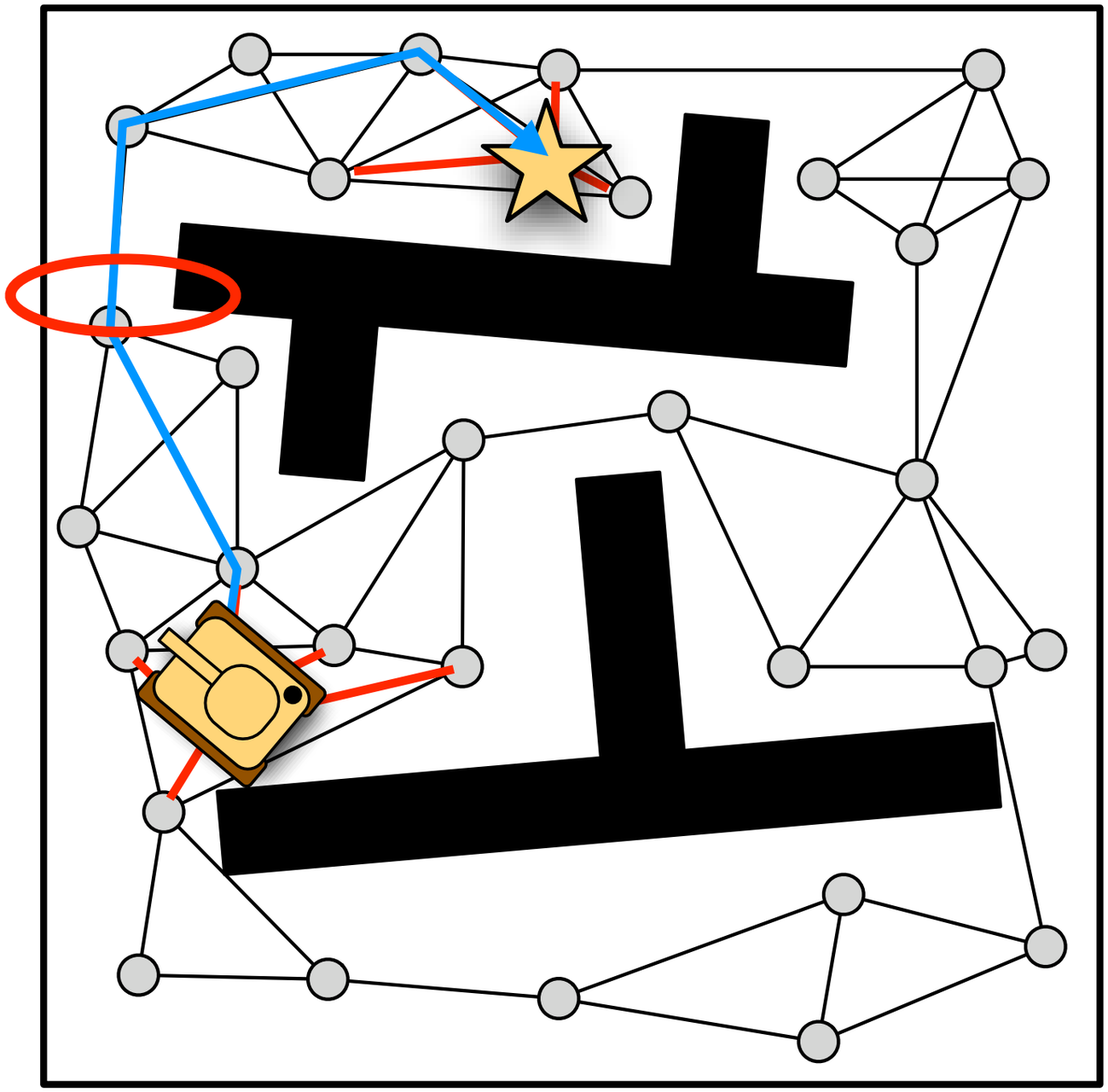
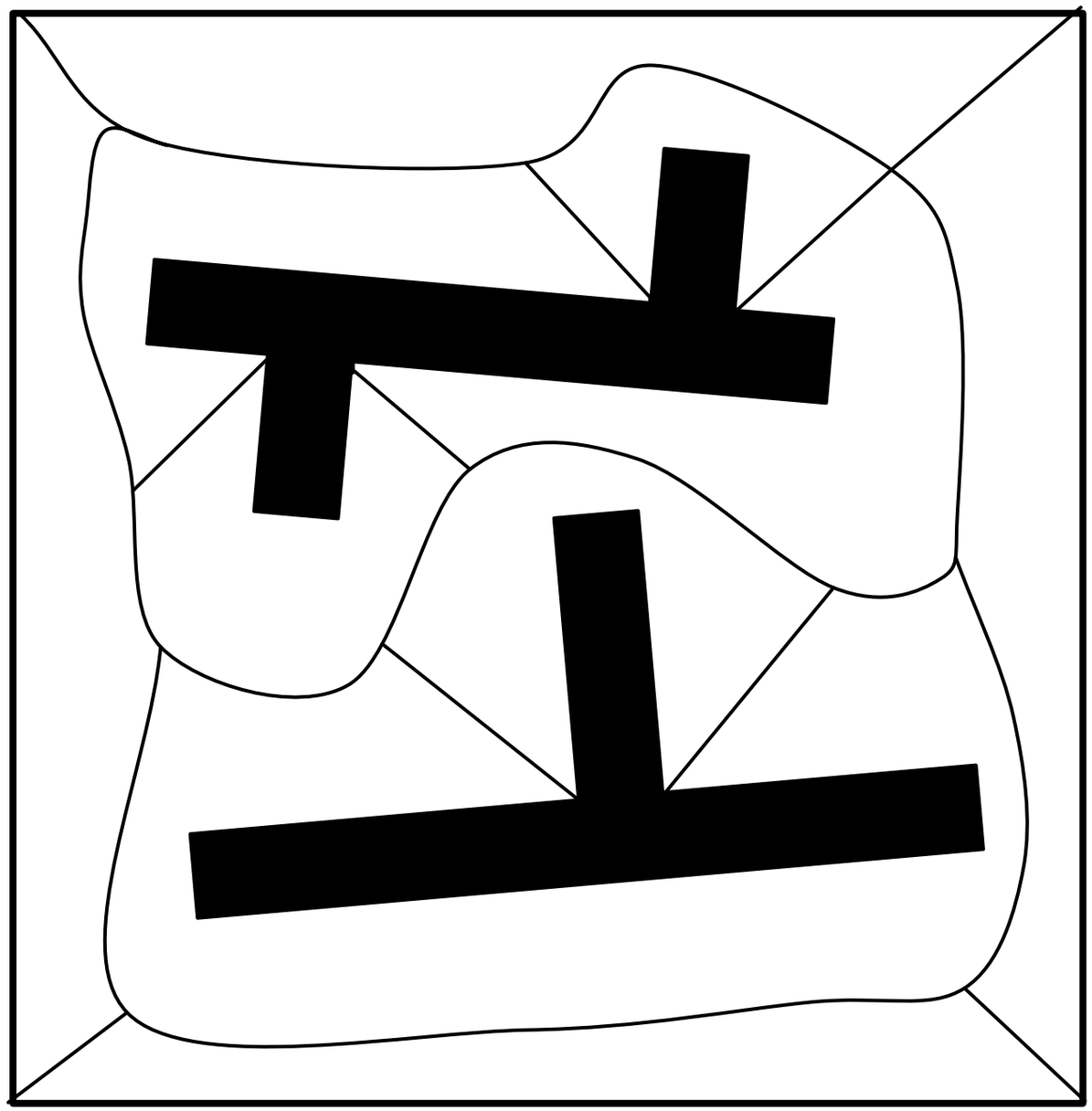


Most PRMs are only useful for agent/configuration used during construction.

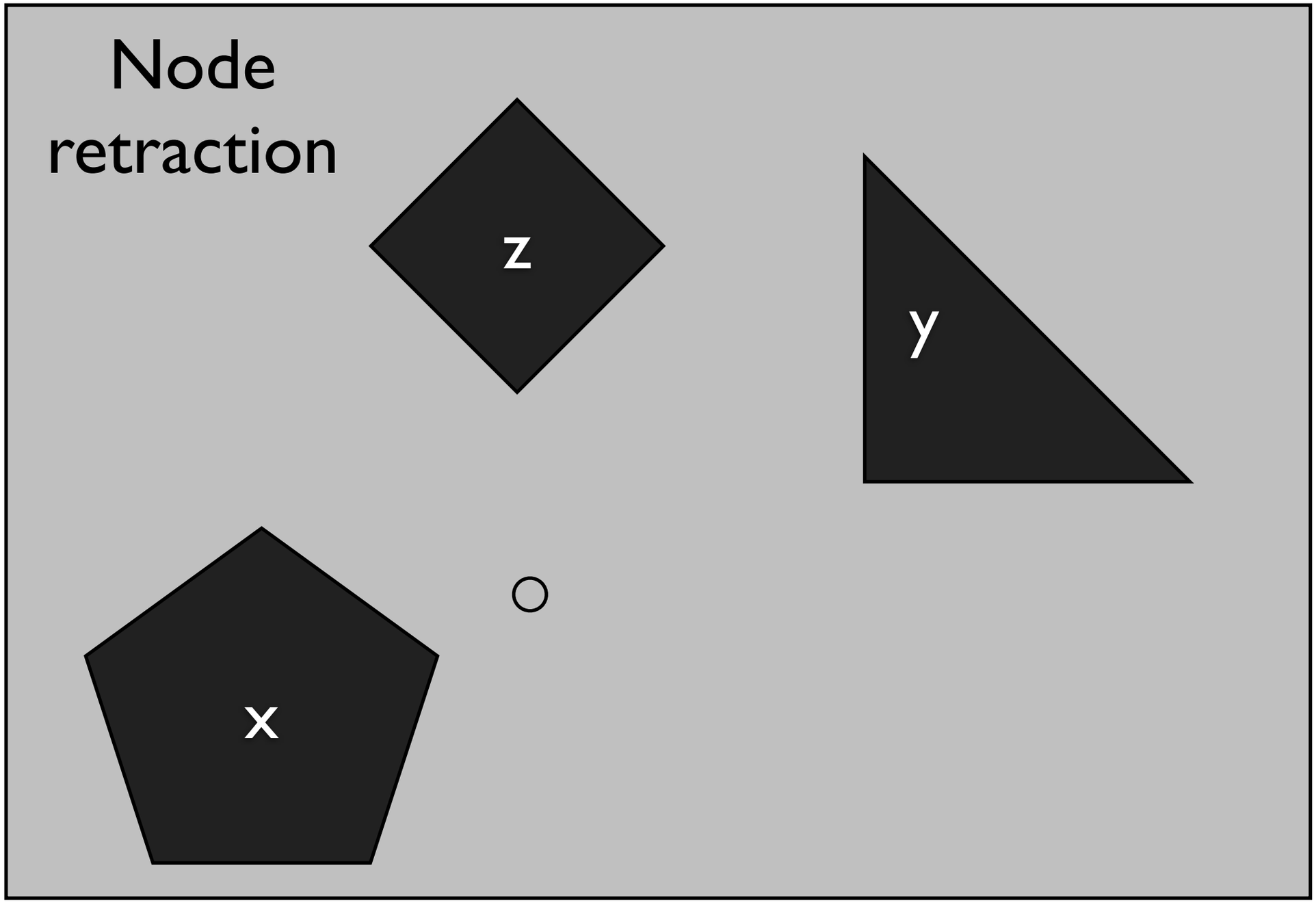
2 options to fix this problem:

- Create a PRM for each unit type
- Opt for a (generalised) Voronoi-based PRM

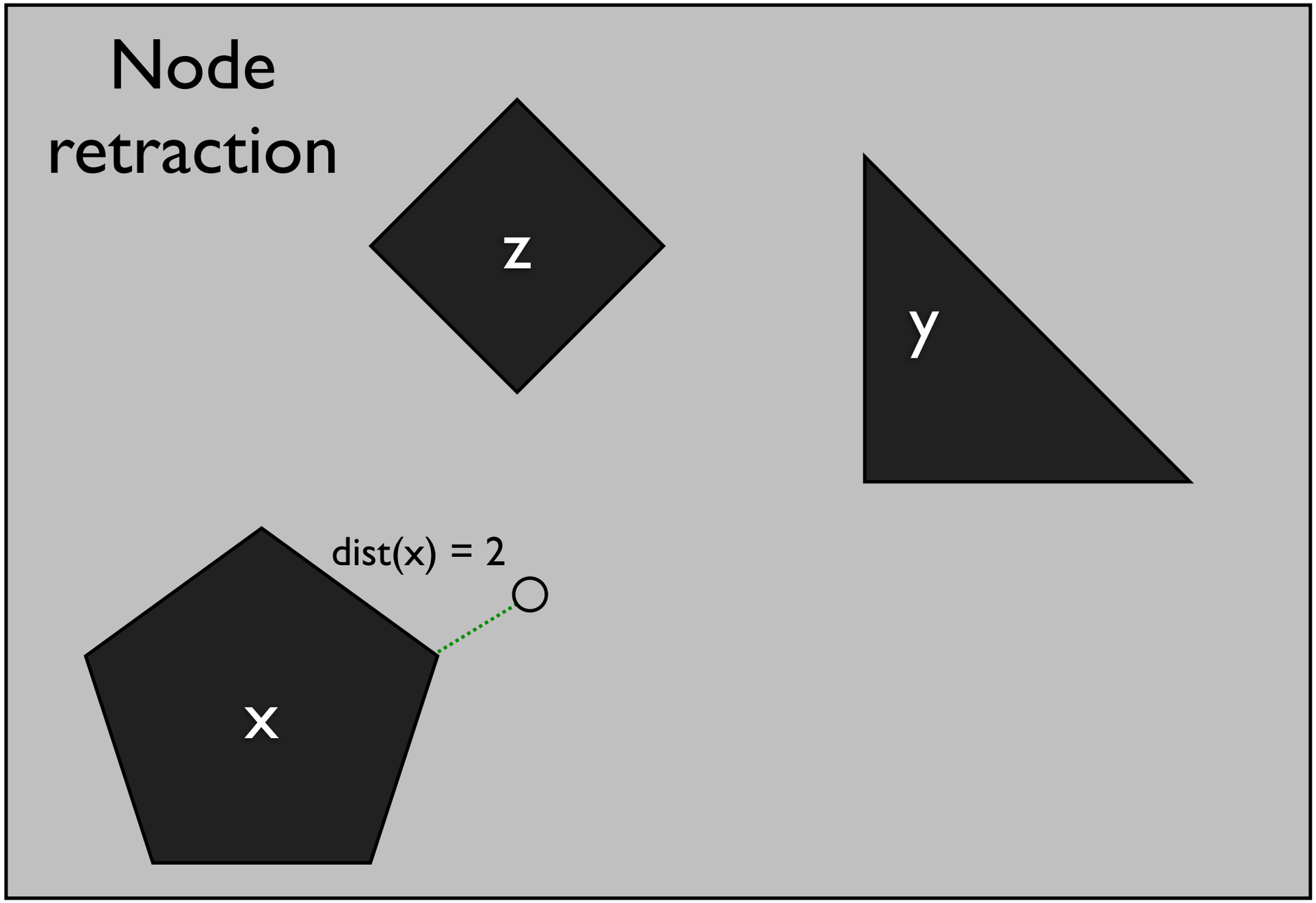


Voronoi diagram: Each point on the medial axis maximises clearance.

Intuition: retract PRM (nodes and edges) to the medial axis.




Identify the closest obstacle then retract the point to a position equidistant to 2 obstacles.

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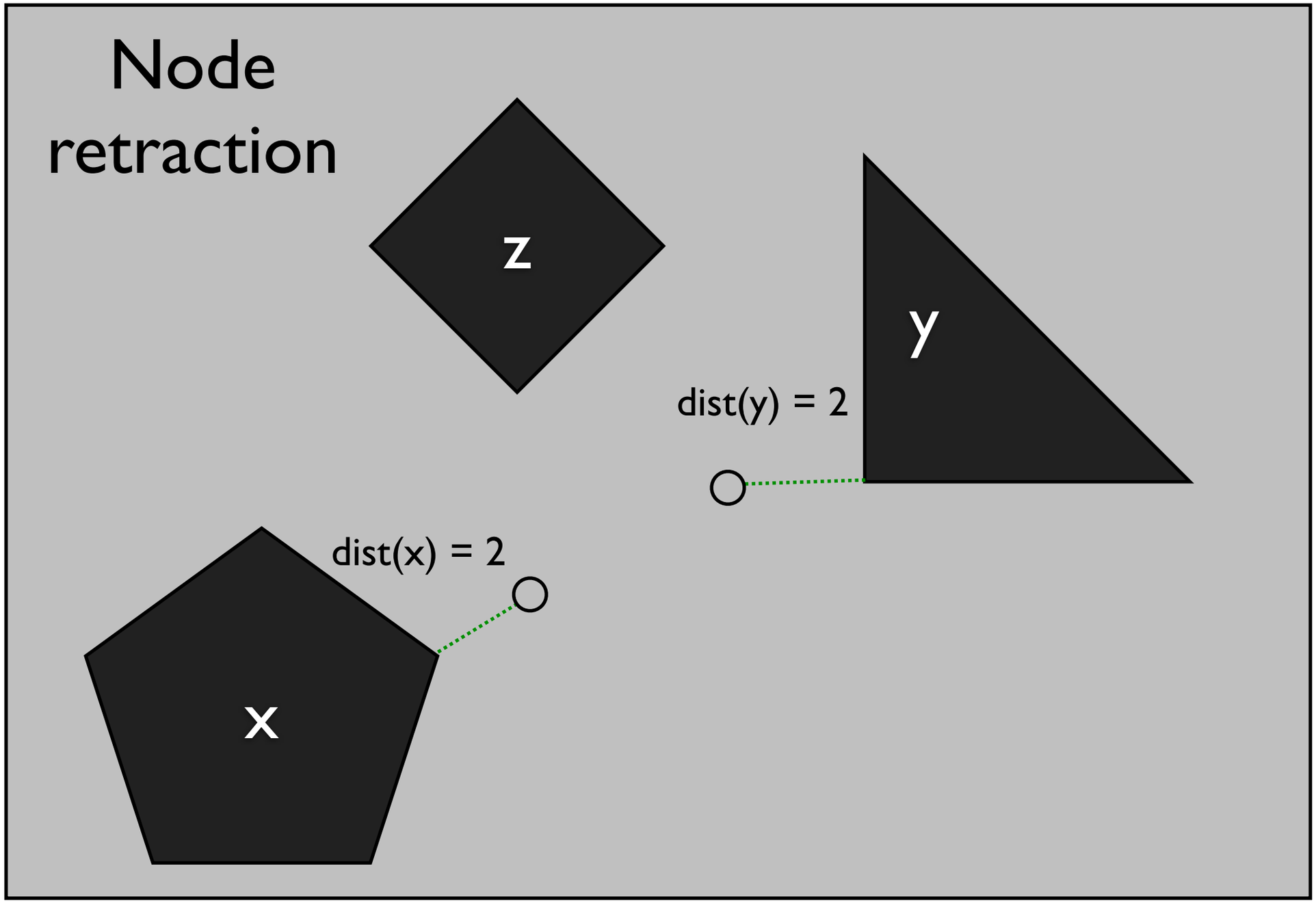


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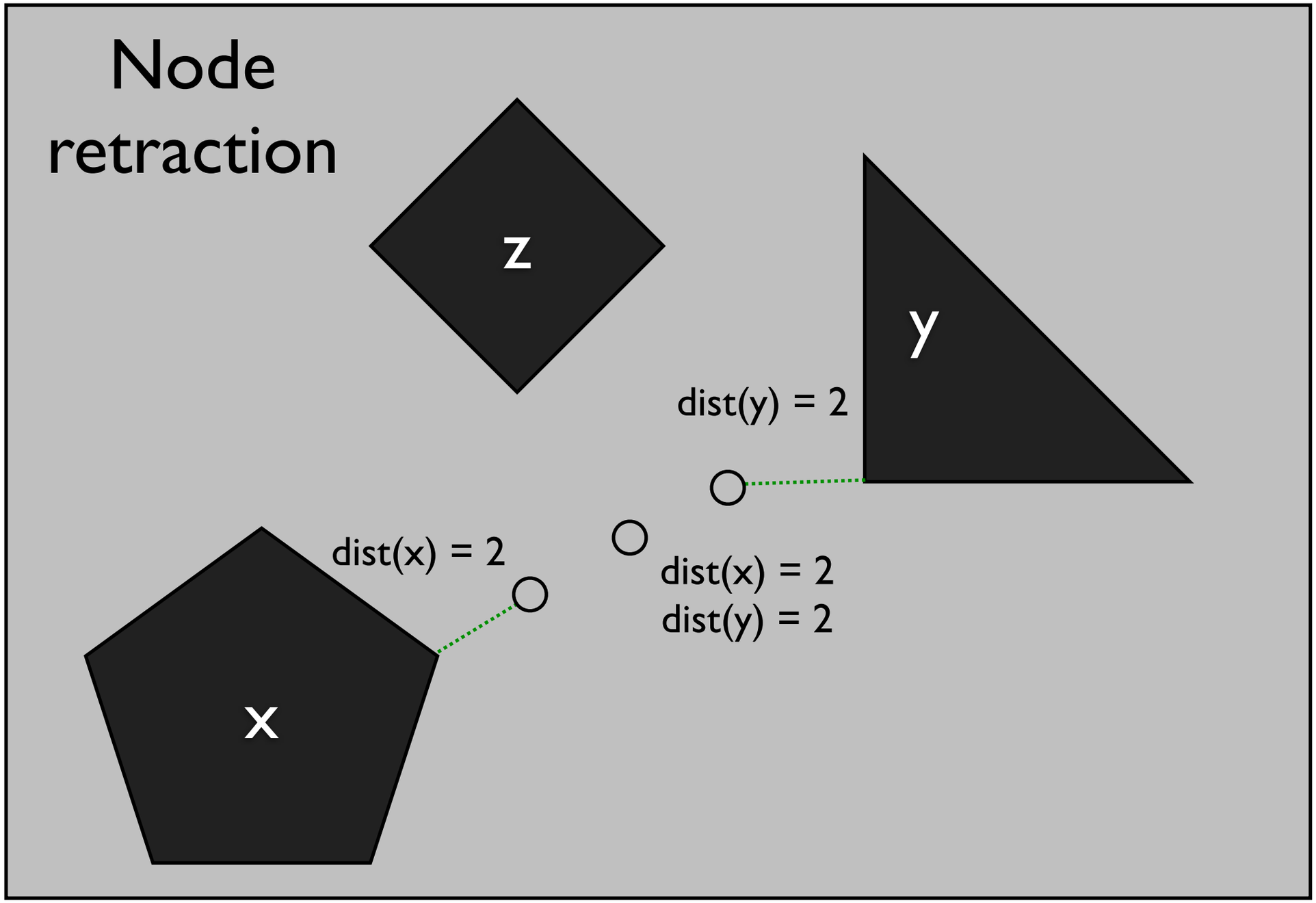


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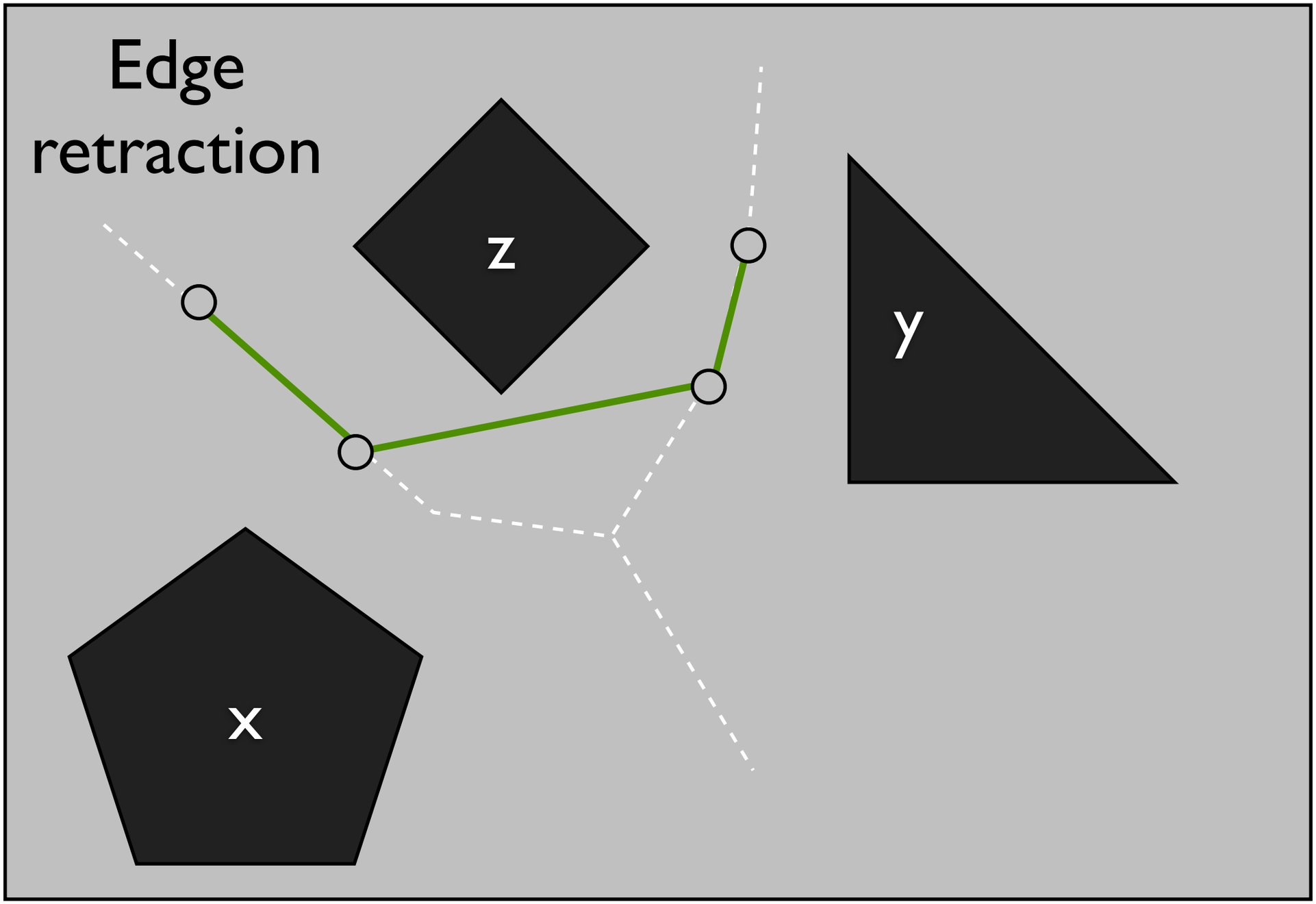
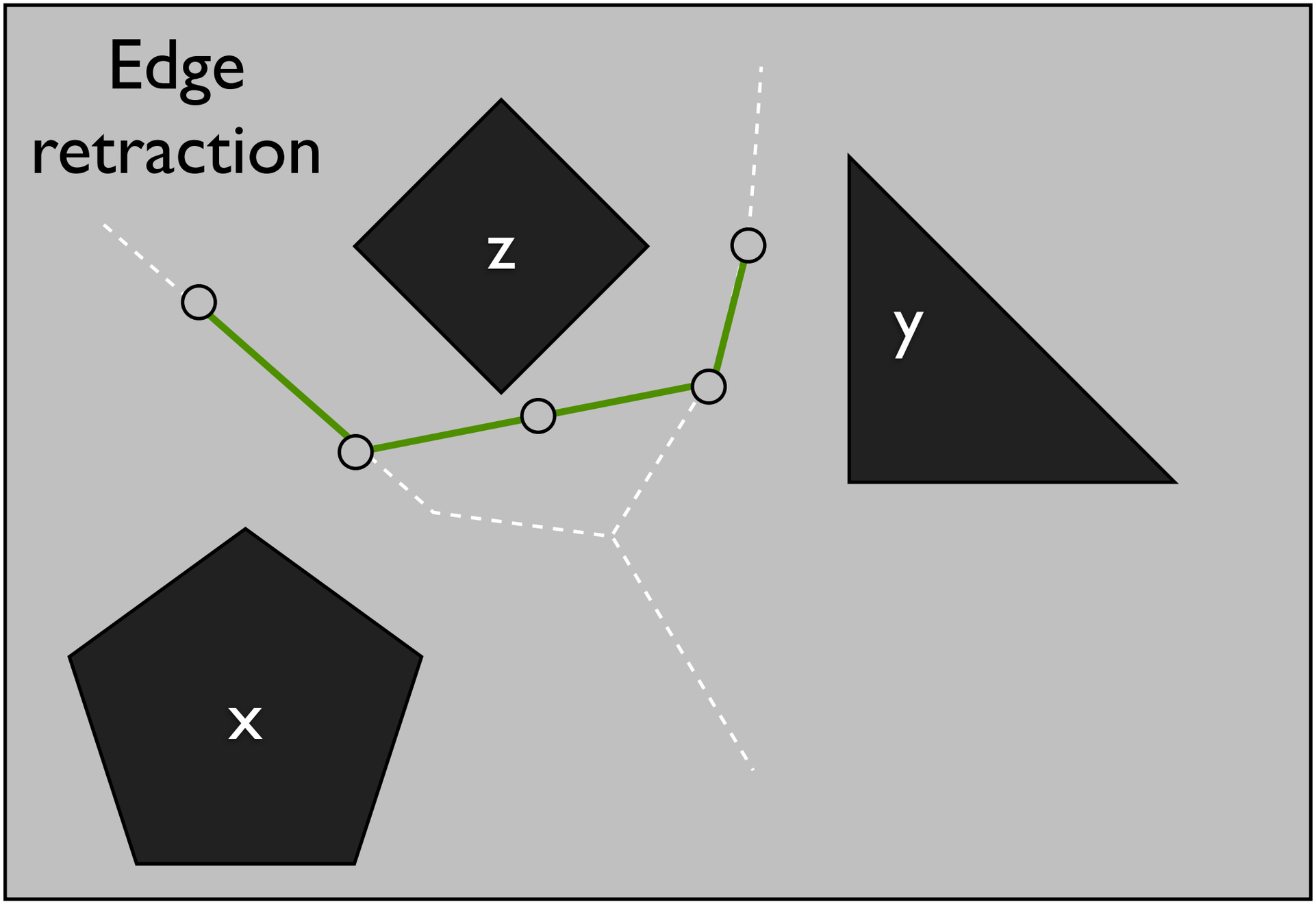



Edges too close to obstacles are retracted by splitting.

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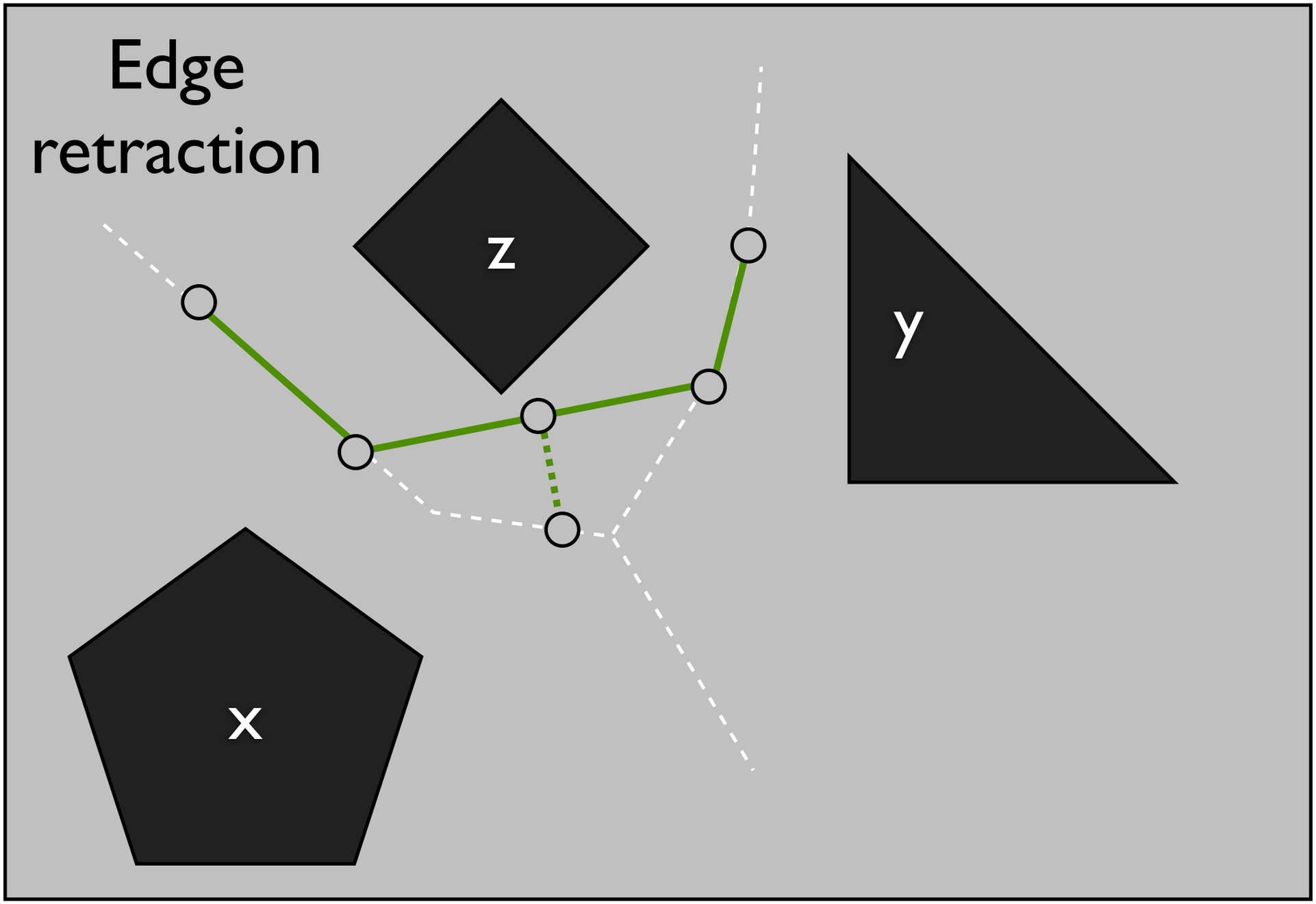


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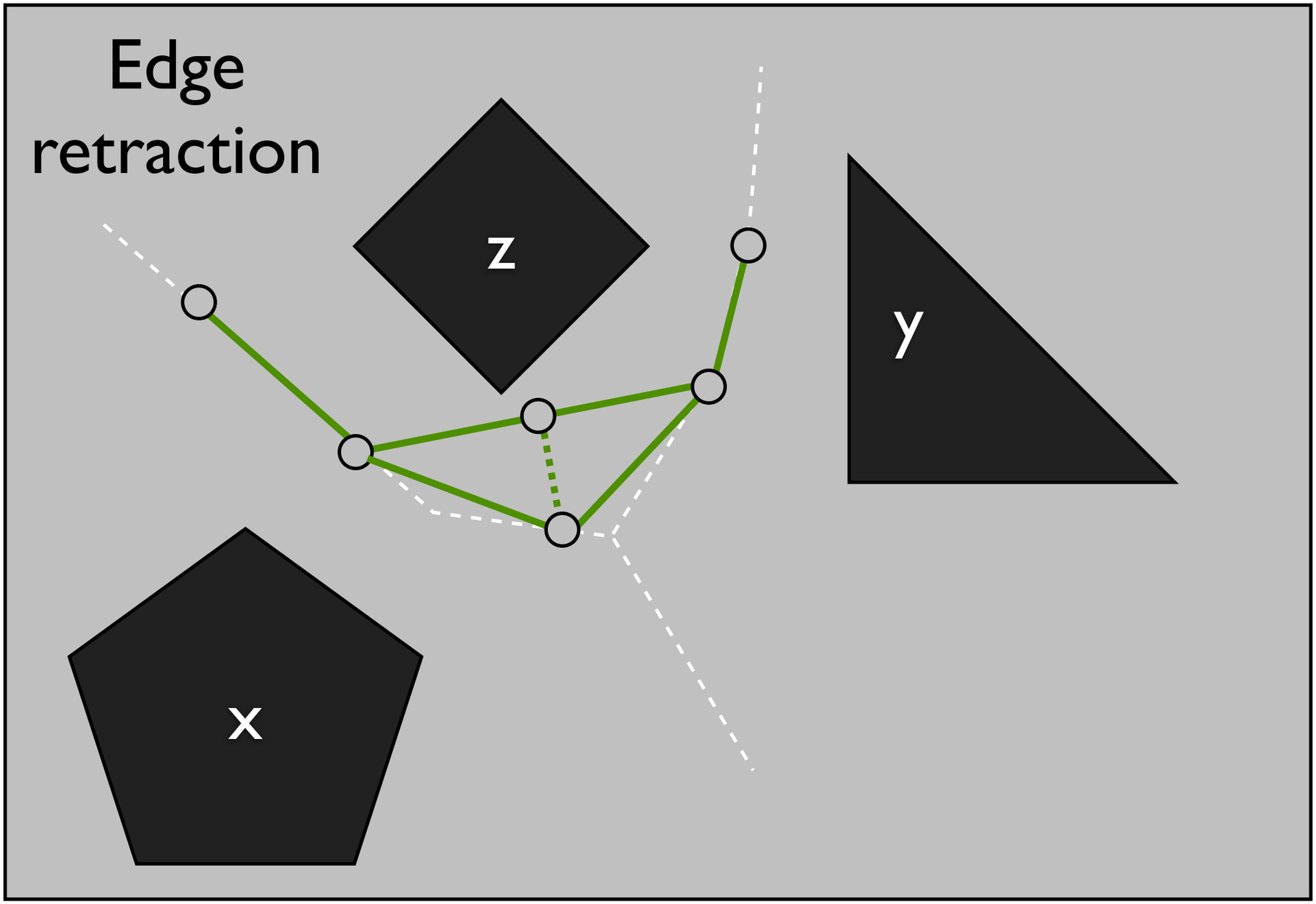
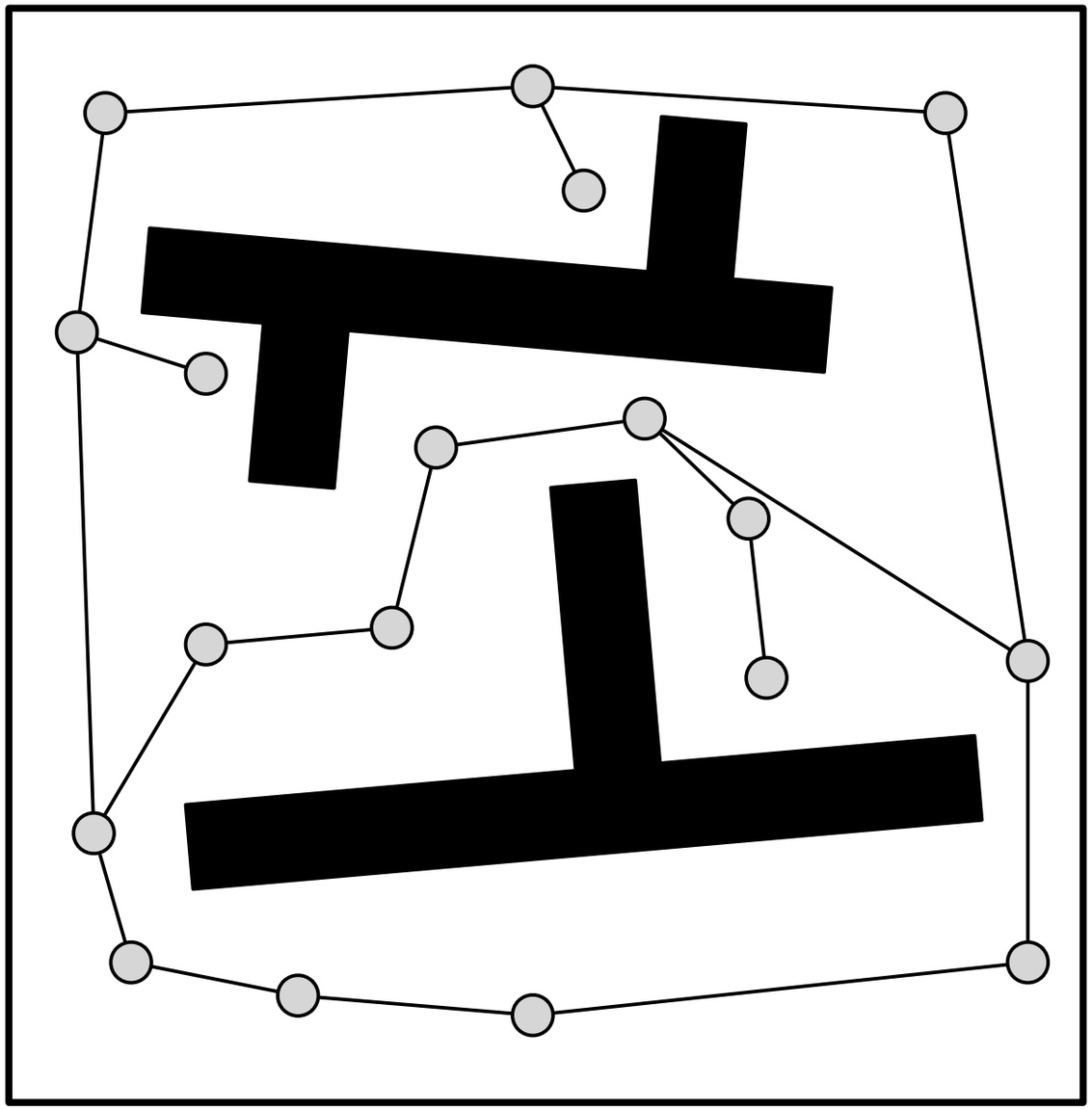


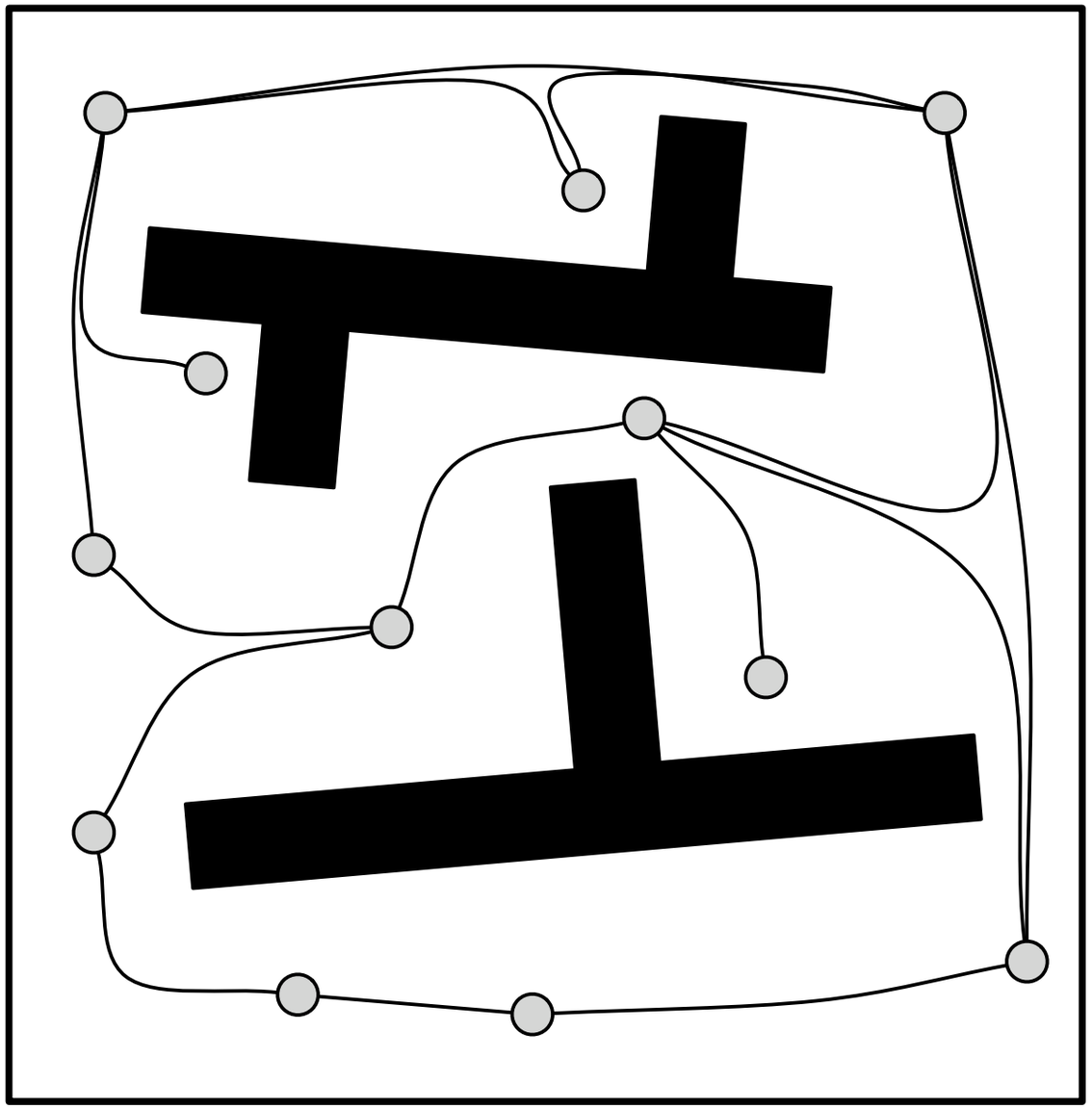

Edges too close to obstacles are retracted by splitting.

- Other improvements:
  - Remove overlapping edges (similar idea to weak dominance)
  - Smooth edges using circular blends

# Other improvements



Remove overlapping edges
(similar to weak dominance)



Apply circular blends

# PRM Advantages

- Fast to construct.
- Low overhead (especially Vornoi PRMs)
- Facilitate pathfinding in n-dimensional spaces
- Generalisable to different unit sizes (Voronoi PRMs only)
- Nice alternative to navigation meshes

# PRM Disadvantages

- Only probabilistically complete*
- Might need several PRMs (to support units with different capabilities).
- Non Voronoi PRMs produce ugly paths.

# Conclusion

- There's more to pathfinding than A*!
- Many open problems; some with nice solutions
  - HPA*, HAA*, Roadmaps
- Pathfinding with abstraction is very effective.

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# Questions?